Non-competing persuaders*

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Abstract

I study Bayesian persuasion games with multiple persuaders in which the persuaders are non-competing: all persuaders want the decision maker to take the same action, regardless of the state. In the case of a single persuader, it is known from previous research that the persuader-optimal information design leaves the decision maker with no surplus. In this paper, I show that with two or more non-competing persuaders and independent tests, there are always equilibria in which the decision maker receives surplus. If there is exogenous noise then the decision maker receives surplus in every symmetric equilibrium, provided the number of persuaders is sufficiently large; asymptotically, the decision maker learns the true state in every Pareto optimal symmetric equilibrium with infinitely many persuaders. Moreover, with sufficient exogenous noise, having more than one persuader not only improves the welfare of the decision maker but also improves the welfare of the persuaders.

Keywords: Bayesian persuasion, endogenous information design, multiple identical persuaders, imperfect information

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1 Introduction

A decision maker who wants to learn about a payoff-relevant state may find herself facing a problematic situation: the only people who are qualified to investigate the true state are biased persuaders who all want her to take a specific action. For example, a government wants to learn whether self-driving cars are safe under the latest technology, so that it can decide whether to legalize autonomous driving on public roads. However, only manufacturers of self-driving cars have the means to test the safety of the technology, and all manufacturers want the government to legalize autonomous driving. A person with nearsightedness wants to know whether a laser surgery can improve her vision. To learn more, she must have her eyes examined at local private eye clinics, but all those clinics profit only when she agrees to have the surgery. A customer needs computer repair services to examine whether she must replace her computer's hardware in order to stop the malfunction, but all the repair services want her to purchase the expensive hardware replacement.

In these cases, the biased persuaders can encourage their wanted action by conducting truthful but biased investigations on the true state. The manufacturers can conduct easy test drives that may not reveal any problem even if the autonomous-driving technology is unsafe. The eye clinics can perform imperfect eye tests which may suggest benefits from the laser surgery even when there aren't any. The computer repair technicians may spend very little effort to test whether alternative solutions to a hardware replacement exist. In fact, if the decision maker (she) asks only one biased persuader (he) of this kind to test the true state for her, the persuader will optimally design a test that increases the expected utility of only himself but not the decision maker (Kamenica and Gentzkow, 2011).

If the information from one persuader, in expectation, does not make the decision maker better off and she can only consult or afford to consult persuaders with the same biased incentive, what can she do to obtain more useful information?

This paper shows that she can simply ask more persuaders to test the true state for her, even if they have an identical objective. When she consults more than one persuader, there always exist equilibria in which the persuaders perform relatively informative tests that strictly benefit the decision maker. If no test can ever really be perfect and there is exogenous type II error (in the sense that the test result sends information against the persuaders' wanted action even if the wanted action is optimal for the decision maker), then the persuaders' tests strictly benefit the decision maker in every symmetric equilibrium, provided the number of persuaders is sufficiently large. Asymptotically, the decision maker learns the true state in every Pareto optimal symmetric equilibrium with infinitely many persuaders.

Moreover, when the probability of the exogenous type II error is sufficiently large, even a persuader prefers the company of other independent persuaders, and their most-preferred (symmetric or asymmetric) test profile strictly benefits the decision maker. In other words, when the decision maker seeks information from more than one persuader, it is a Pareto improvement for both herself and the persuaders.

Why does an increase in the number of persuaders improve the information outcome for the decision maker? If all persuaders are identical, why don't they simply mimic the strategy of a single persuader and conduct manipulative tests that leave no surplus to the decision maker? The answer is that they switch to more informative tests because of a cooperative motive: by increasing the credibility of a positive result from his own test, a persuader can help offset potential negative results from the other persuaders, which is desirable for all persuaders as a team. In equilibrium, this incentive to improve the credibility of positive results leads to positive surplus for the decision maker, and the ability to offset negative results leads to positive surplus for the persuaders. The next section elaborates this intuition with a detailed example.

The results in this paper make a novel contribution to the theory of endogenous information design. As mentioned earlier, when only one biased persuader tests the true state, his optimal test design leaves no surplus for the decision maker (Kamenica and Gentzkow, 2011). This implies that if there are multiple persuaders who (a) are non-competitive in the sense that all persuaders want the decision maker to take the same action, regardless of the state, and (b) can correlate the outcomes of their information designs perfectly, then it will be optimal for persuaders to behave collectively like a single persuader, leaving the decision maker with no surplus. Therefore, if the decision maker is to get positive surplus with multiple persuaders, some deviation from either (a) or (b), or both, is necessary.

Gentzkow and Kamenica (2017a, 2017b) investigate environments in which (b) holds but (a) is violated: persuaders can correlate the outcomes of their information designs perfectly but persuaders are competitive in the sense that they have different preferences over the action, conditional on the state. For such environments, Gentzkow and Kamenica show that if any persuader can unilaterally deviate to induce any feasible information outcome that is more informative, then having multiple persuaders improves the decision maker's utility. The intuition is that, when persuaders have different objectives, they are incentivized to collect extra information in order to induce their own preferred outcomes, and this extra information gives surplus to the decision maker.

This paper examines the polar opposite case in which (a) holds but (b) is violated: persuaders are non-competitive but are constrained to choose information designs that generate conditionally independent outcomes. By focusing on such environments which are free of competition, regulation, or concerns of reputation, this paper uncovers a new intuition that rationalizes positive surplus for the decision maker in a multi-persuader game from a different, cooperative angle. A persuader reveals extra information to help his fellow persuaders. He decreases his production of false but favorable information in an unfavorable state, so that favorable information from him is convincing enough to outweigh unfavorable information from other persuaders. This incentive is particularly strong when unfavorable information cannot be perfectly avoided due to exogenous errors, and it makes the persuaders deviate from information designs with frequent false-favorable outcomes and no surplus for the decision maker.

Other studies related to this paper include two papers by Li and Norman that extend Gentzkow and Kamenica (2017a). Li and Norman (2018) look at simultaneous games of independent persuaders with different preferences. The paper provides an example in which two persuaders release less information than one. Li and Norman (2017) look at arbitrarily correlated persuaders with different preferences who choose their tests in a sequence. It shows that adding an extra persuader at the end or in the middle of the sequence can result in an information loss. These papers point out that, when persuaders have different preferences, it can be worse for the decision maker to consult a second persuader. In my paper, adding a second persuader simultaneously or sequentially never hurts the decision maker because her payoff is already at the lowest with one persuader.

Other papers that discuss competitive persuasion include Board and Lu (2018) and Au and Kawai (2019). Board and Lu (2018) study competing sellers of the same product who try to attract searching buyers by disclosing information about the product. They show that the effect of competition on information disclosure depends on whether the buyers' beliefs are private. Au and Kawai (2019) study the competition between sellers of different products who try to attract a single buyer by disclosing information about their own products. The effect of competition is ambiguous in general.

On the topic of noisy tests, a paper by Rick (2013) studies a one-persuader game with exogenous noise. He shows that if the persuader cannot choose the test design but can repeat the test an arbitrary number of times and report only the final result, then exogenous false-positive errors can benefit the decision maker. This is because the error induces a posterior belief that the persuader favors even when he has not exerted effort to harvest false-positive evidence. This gives the persuader an incentive to reduce false-positive reports and deliver more-truthful information. In this paper, because the persuaders are able to choose the test design, the intuition from Rick (2013) does not apply. When the accuracy of feasible test designs is lowered due to exogenous noise, a persuader releases less information if he is the only persuader. Persuaders release more information only when there exist other persuaders. They are incentivized to design informative tests because they want to make their own positive results powerful enough to outweigh others' negative results. This incentive disappears if a persuader is alone.

There are papers (e.g., Battaglini, 2002; Ambrus and Takahashi, 2008; Ambrus and Lu, 2014) on cheap talk persuasion games with multiple persuaders. However, note that in the binary-state, binary-action game studied in this paper, if the decision maker observes only the results of the tests and not the design, the only equilibrium is a trivial one in which the persuaders always conduct completely uninformative tests with uniformly positive results. The decision maker is never persuaded (Sobel, 2013). Therefore, in this paper, it is crucial that the decision maker observes both the design and the outcome of the test.

Other papers (e.g., Bhattacharya and Mukherjee, 2013; Felgenhauer and Schulte, 2014; Hart, Kremer, and Perry, 2017) study persuasion games in which state-independent persuaders cannot choose the test design but can hide unfavorable test results. A key distinction is that the persuaders in those papers decide whether to report a result only after they see the results, whereas the persuaders in this paper unconditionally commit to report all results. Therefore, the persuaders in those settings report only favorable evidence, and they avoid tests that can yield negative results in a favorable state because fewer favorable reports means lower posterior belief for the decision maker. In contrast, in this paper, there are benefits to choosing tests that yield negative results in a favorable state. The possibility of false-negative results increases the decision maker's posterior belief when she sees negative results, which decreases the number of positive results needed to persuade the decision maker.

Many assumptions in this paper are similar to those in the standard voting literature, such as Feddersen and Pesendorfer (1998), but there is one crucial difference that leads to very different results. The decision maker in this paper does not commit to any decision rule that is based only on test results. In the voting literature, the decision maker takes a certain action if the number of votes passes an exogenous threshold, regardless of the voting strategy (e.g., the unanimity rule or the majority rule). In contrast, the decision maker in this paper chooses an endogenous decision rule that changes with the test designs of all persuaders. In this sense, a persuader is always pivotal regardless of the other persuaders' test results. Contrary to the voting game, if the decision maker were to commit to an exogenous result-based rule (e.g., two positive results out of three tests), the persuaders would simply choose uninformative tests that always yield positive results. If that were the case, the decision maker would rather ignore the persuaders and always choose the default action. This outcome is undesirable for both the decision maker and the persuaders.

The remainder of the paper is organized as follows. Section 2 illustrates the paper's results and intuition through an example. Section 3 formally introduces the model and proves the main results. Section 4 discusses the robustness of the main results with respect to alternative modeling choices. Section 5 concludes the findings. All proofs can be found in the Appendix.

2 Example

A government is considering whether to legalize the use of autonomous, self-driving cars on public roads. However, it is unsure whether autonomous driving is safe under the latest technology. The government's payoff function is described by the table below. It is willing to legalize autonomous driving on public roads if and only if the probability that the technology is safe is at least 0.99. Without additional information, the common prior belief is that Pr(safe) $=$ Pr(unsafe) $= 0.5$. Its default action is not to legalize (status quo) and its payoff is 0.

Government's payoff

		True state: Is the technology safe?	
		safe	unsafe
Action	legalize	1	-99
	not legalize	0	0

To make a responsible decision, the government decides to learn more about the true state. However, only the manufacturers of autonomous cars have the means to test the safety of the technology, and these manufacturers have a different incentive: they all want the government to legalize autonomous driving, regardless of its safety. Specifically, suppose that there are a number of manufacturers whose cars are all equipped with the same state-of-the-art autonomous-driving technology. If the government legalizes the use of these cars on public roads, every manufacturer's payoff is 1; otherwise, every manufacturer's payoff is 0.

To gather information on the safety of the latest autonomous-driving technology, the government permits n of these manufacturers to test their cars on public roads. Each manufacturer's test independently yields either a positive result (e.g., no accident) or a negative result (e.g., accident occurs). Prior to the public road test, a manufacturer is not certain about the safety of the technology, either, but it can manipulate the details of the test design such as time, location, and test routes so that it essentially chooses the conditional probabilities $x = \Pr(\text{positive result}|\text{not safe})$ and $y = \Pr(\text{positive result}|\text{safe})$. Moreover, because road conditions are unpredictable, it is possible that an accident occurs even if the technology is safe. That is, there exists an upper bound $\bar{y} < 1$ on the true-positive probability such that $y \leq \bar{y}$ for all feasible test designs.

The government observes both the test designs and the test results of the n manufacturers. It updates its belief based on this information and makes a decision that best matches its posterior belief about the safety of autonomous driving under the latest technology.

Suppose that $n = 1$ - that is, the government permits only one manufacturer to test drive on public roads. Then, the government's ex-ante expected utility is always 0 regardless of $\bar{y}$; it does not benefit from the manufacturer's information. This is because the manufacturer optimally designs an easy-to-pass test that can sometimes yield a positive result even if the technology is unsafe. In equilibrium, the government either chooses the default action of not legalizing when the test result is negative, or is just indifferent between legalizing and not legalizing when the test result is positive. Its ex-ante expected utility before seeing the test result is the same as if it had chosen not to see the result and never pass the law.

If the information from one manufacturer is ex-ante useless, and all manufacturers are exactly the same, what can the government do to improve its welfare? In particular, if the government does not have the expertise or the budget to conduct better tests itself, how can it learn more about the safety of the autonomous driving technology?

The answer is that the government can simply permit more manufacturers to test drive on public roads. This solution barely costs the government anything but is highly effective: when the government allows multiple manufacturers to test drive on public roads, they design better tests that benefit the government. Moreover, if $\bar{y}$ is sufficiently low, the manufacturers also benefit, as they expect the government to legalize autonomous driving with a higher probability when $n > 1$. In other words, issuing the test-drive permit to multiple manufacturers is a Pareto improvement that benefits both the government and the manufacturers. Below, I illustrate the intuition with a numerical example.

Suppose that even if the technology is safe, there is a 1% chance that some accident will happen during the most revealing test drive (i.e., $\bar{y} = 0.99$). If $n = 1$, then the single manufacturer chooses the false-positive and true-positive probabilities to be $(x_{solo}, \bar{y}) = (0.01, 0.99)$. As mentioned earlier, the government legalizes autonomous driving if and only if the test is successful, but its ex-ante expected payoff is 0 because it never strictly prefers to legalize.

If $n = 2$, there are two possible symmetric test designs for the two manufacturers: they can either both choose an easy test with a high false-positive probability such that the government passes the law only when both tests are successful, or both choose a difficult test with a low false-positive probability such that the government passes the law as long as one test is successful. In the former case, the test is $(x_h, \bar{y}) = (0.0995, 0.99)$; in the latter case, the test is $(x_l, \bar{y}) = (0.0001, 0.99)$. The latter test is strictly more informative than the former.

If both manufacturers choose the easy test $(x_h, \bar{y})$ then the game outcome resembles the single-manufacturer outcome: the government never strictly prefers to legalize (it is indifferent when both test results are positive) and its ex-ante expected payoff is 0. If both manufacturers choose the more informative test $(x_l, \bar{y})$, then the government is indifferent when only one test result is positive, but it strictly prefers to legalize when both results are positive. As a result, the government's ex-ante expected payoff is 0.49. This is a great improvement compared to the single-manufacturer outcome (expected payoff = 0), considering that the government's maximum possible ex-ante expected payoff is just 0.5 (which is obtained when it learns the true state). Hereafter, I call a test profile "*beneficial*" if it induces a strictly positive ex-ante expected payoff for the government, and "*non-beneficial*" if the induced ex-ante expected payoff for the government is zero.

Between the two possible test choices, which is more likely? As it turns out, the non-beneficial test profile $(x_h, \bar{y})$ does not constitute an equilibrium. The beneficial test profile $(x_l, \bar{y})$ does. Why is the non-beneficial test profile unstable? What can be a profitable deviation? The key is to understand the manufacturer's trade-off between the probability of positive test result and the ex-post legalization standard from the government. The non-beneficial test $(x_h, \bar{y})$ generates a positive result more often, but the government's ex-post standard for legalizing autonomous driving is high (it requires to see two positive results);

the beneficial test $(x_l, \bar{y})$ generates a positive result less often, but the government's ex-post standard is low (it requires only one positive result). When accidents cannot be completely avoided regardless of the test design even if the technology is safe, a high standard from the government is very undesirable for the manufacturers. Therefore, if manufacturer 1 chooses the easy test $(x_h, \bar{y})$, the best response of manufacturer 2 is, in fact, to choose a difficult test $\left(x^{'}, \bar{y}\right) = (0.000111, 0.99)$. The lower false-positive probability $x^{'}$ is chosen such that if this test result is positive, the government legalizes autonomous driving even if manufacturer 1's test result is negative.

The paper generalizes this result and shows that, fixing any $n > 1$, non-beneficial tests are not chosen in any symmetric equilibrium if $\bar{y}$ is sufficiently low; fixing any $\bar{y} < 1$, non-beneficial tests are not chosen in any symmetric equilibrium if n is sufficiently high. In contrast, there is always a symmetric equilibrium in which the manufacturers choose very informative, beneficial tests such that the government legalizes autonomous driving as long as one test result is positive. In these equilibria, when the government permits more manufacturers to test drive on public roads, each manufacturer strictly increases the informativeness of its individual test drive. Asymptotically, as the number of permitted manufacturers goes to infinity, the government learns the true state in any Pareto optimal symmetric equilibrium.

Now, let's return to the case of two manufacturers and have a look at the manufacturers' ex-ante expected payoff. Compared to the case of one manufacturer, is the government more or less likely to legalize autonomous driving? If the two manufacturers can somehow coordinate to choose their most preferred pair of tests, will they indeed both conduct beneficial tests, or will one simply waste a permit and do nothing?

In the equilibrium where the two manufacturers both choose the beneficial test $(x_l, \bar{y})$, the government legalizes autonomous driving with probability 0.50005. This, in fact, is uniquely the highest probability that any pair of feasible tests can achieve. In comparison, if one manufacturer performs the optimal single-manufacturer test $(x_{solo}, \bar{y})$ while the other wastes the permit and does nothing, the government legalizes autonomous driving with probability 0.5. This probability is lower because the government never tolerates a negative test result in the case of a single informative manufacturer, but it tolerates one negative test result in the case of two manufacturers who choose $(x_l, \bar{y})$. Because the probability of legalization is the manufacturers' ex-ante expected payoff, it is in the manufacturers' best interest to both perform the beneficial test $(x_l, \bar{y})$. Permitting two rather than one manufacturer to test drive on public roads not only improves the government's welfare but also improves the manufacturers' welfare. After issuing permits to both manufacturers, the government does not need to spend any extra resource to supervise their test drives, because the manufacturers are self-motivated to choose tests that benefit the government.

The paper generalizes this result and shows that, when $\bar{y}$ is sufficiently low, the manufacturers' payoff is higher in a multi-manufacturer equilibrium with beneficial tests than in the single-manufacturer equilibrium with the non-beneficial test. Moreover, their payoff in the single-manufacturer equilibrium is the upper bound of their payoff when they choose a non-beneficial test profile. Therefore, this implies that the manufacturers' most preferred test profile must benefit the government.

Finally, to understand the role of $\bar{y}$, one may wonder what the equilibrium outcome is if there is no upper bound on the true-positive probability y. If $y = 1$ is feasible, then for any $n \geq 2$, beneficial and non-beneficial equilibria for the government co-exist. However, the manufacturers' expected payoff is maximized in equilibria that do not benefit the government. When accidents can be perfectly avoided by test design, it is no longer attractive for the manufacturers to sacrifice the probability of positive results in order to have the government tolerate negative test results. Therefore, in these non-beneficial equilibria, the manufacturers design relatively high false-positive probabilities in their tests and the government legalizes autonomous driving only when all test results are positive. This shows that exogenous false-negative errors, which are commonly observed in reality, are important to induce beneficial outcomes for the government. These exogenous errors incentivize a manufacturer to choose a highly informative test so that its positive result is convincing enough to offset potential negative results from the other manufacturers.

3 General model

In this section, I study a general case with n identical, non-competing persuaders. Section 3.1 sets up the model. Section 3.2 derives the necessary and sufficient conditions for an equilibrium that benefits the decision maker. It also shows that if exogenous noise is absent in the environment, beneficial equilibria for the decision maker always exist, but the persuaders maximize their payoff in an equilibrium that does not benefit the decision maker. Based on these results, the sections 3.3-3.5 focus on environments with exogenous noise to identify cases in which equilibria benefiting the decision maker are the only or the most likely game result.

Sections 3.3 and 3.4 focus on only symmetric equilibria and deliver the first set of results: the decision maker benefits from every symmetric equilibrium when there is sufficient exogenous noise or when the number of persuaders is sufficiently large. With infinitely many persuaders, the decision maker learns the true state with probability 1 in every Pareto optimal symmetric equilibrium.

To justify symmetry as a reasonable equilibrium selection criterion, recall that the persuaders are identical and independent. An asymmetric equilibrium requires that these independent persuaders have some ability to coordinate and play different strategies, which could be too demanding, especially when the number of persuaders is large. Take the motivating story for example. There is always an asymmetric equilibrium in which only one manufacturer test drives while the others do nothing. In reality, this means that only one manufacturer gets to be a "good citizen" and provide information to the government, while the other $n-1$ manufacturers simply deny the government's request. One can imagine that identical, independent manufacturers might disagree on who this good citizen should be, thus making this asymmetric equilibrium difficult to play. In other scenarios such as the doctor-patient and technician-customer examples mentioned in the Introduction, the persuaders may not know each other's identity, making it even more difficult to coordinate on an asymmetric strategy profile.

Nevertheless, this paper is not silent on asymmetric equilibrium. Section 3.5 studies all equilibria in general and delivers the second set of results: when there is sufficient exogenous noise, a persuader is better off when he is not the only one who conducts an informative test, and the persuader-optimal equilibrium in a multi-persuader game is one that benefits the decision maker. Therefore, having multiple persuaders increases the welfare of all players in the game.

3.1 Setup

There are two states of the world: $\omega \in \{H, L\}$. [1] There are n persuaders (he) and a decision maker (she). The decision maker can choose one of two actions, a_H or a_L. (Think of a_H as "legalize autonomous driving" and a_L as "not legalize" in the motivating example.) Her preference is described by a utility function u that depends on her action and the true state, as illustrated in the table below, for some $p_d \in \left(\frac{1}{2}, 1\right)$. [2]

[1] The main result of the paper is robust when the state space is a continuum; see discussion in Section 4.

[2] While this paper normalizes the decision maker's payoff associated with a_L (status quo) to zero, this paper applies to a more general class of preferences. As long as there exists a threshold $p_d \in \left(\frac{1}{2}, 1\right)$ such that the decision maker prefers a_H if and only if $\Pr(H) \geq p_d$, all results in this paper hold.

Decision maker's payoff

		state	
		H	L
action	a_H	1	$-\frac{p_d}{1-p_d}$
	a_L	0	0

With these preferences, the decision maker prefers a_H if and only if the posterior probability for state H is above p_d. Thus, p_d can be viewed as the decision maker's "threshold of doubt." I assume here that the decision maker chooses a_H when she is indifferent.

The persuaders, on the other hand, all prefer that the decision maker chooses a_H regardless of the true state. Their preference can be represented by a common state-independent utility function v with $v(a_H) = 1$ and $v(a_L) = 0$.

The persuaders and the decision maker share a common prior: $\Pr(H) = \Pr(L) = \frac{1}{2}$.[3] Each persuader i designs an endogenous test on the true state. A test is a garbling of the true state that generates a message $m_i \in \{positive, negative\}$ with probabilities conditional on ω. Results of this paper are not qualitatively affected by the binary-message assumption.[4] The strategy of persuader i is to choose the conditional probabilities (x_i, y_i), where $x_i \equiv \Pr(positive|L)$ and $y_i \equiv \Pr(positive|H)$. Assume that $x_i \leq y_i$ for all i so that "positive" is positively associated with state H.

y_i may be bounded from above due to an exogenous probability of false-negative results. Let $\bar{y} \in (0, 1]$ denote the upper bound. Then, $y_i \leq \bar{y}$ for all i. In contrast, I assume that x_i is unbounded from below.[5]

All persuaders choose their tests simultaneously. The decision maker observes both the tests $((x_1, y_1), ..., (x_n, y_n))$ and their results $(m_1, ..., m_n)$.

The timeline of the game is summarized below.

1. N persuaders simultaneously design tests $(x_1, y_1), ..., (x_n, y_n)$.

2. Nature chooses the state of the world.

3. Each test generates a result m_i.

[3] The assumption that $\Pr(H) = \Pr(L) = \frac{1}{2}$ is without loss of generality. See Section 4 for details.

[4] Results of this paper hold when the message space is larger. See Section 4 for details.

[5] To persuade the decision maker that the true state is likely H, the persuaders naturally have the incentive to endogenously choose a relatively high x_i. Therefore, it is not restrictive to assume that x_i can be arbitrarily low because, unlike an upper bound on y_i, a small exogenous lower bound on x_i does not bind. Section 4 discusses this in detail.

4. After observing the test designs and the test results, the decision maker Bayesian updates her belief about the true state and chooses an action a.

In this paper, I let U denote the ex-ante expected utility of the decision maker before test results are revealed and V denote the ex-ante expected utility of each persuader. Note that, when the decision maker receives no information from any persuader, her expected utility is 0; in this case, she always chooses a_L. Therefore, U is always non-negative because the decision maker can always guarantee an ex-ante expected utility of at least 0 by choosing a_L unconditionally. In any event, the highest possible value of U is $\bar{U} \equiv \frac{1}{2}$, which occurs when the decision maker learns the true state.

In the remainder of the paper, I use the term "*beneficial*" to describe the case in which $U > 0$, i.e., the persuaders' information leaves the decision maker with a positive surplus, and "*non-beneficial*" if $U = 0$.

Definition 1. An equilibrium is *"beneficial"* if and only if the decision maker's ex-ante expected utility is some $U > 0$. Otherwise, the equilibrium is *"non-beneficial"*.

The solution concept used in this paper is strict perfect Bayesian equilibrium. The requirement of strictness is that the best response of each persuader must be unique. This eliminates the "nuisance" equilibrium when $\bar{y} = 1$ such that every persuader chooses the fully-revealing test.[6]

3.2 Sufficient and necessary condition for beneficial equilibria

In this section, I identify the necessary and sufficient condition for an equilibrium that benefits the decision maker. Based on this condition, I provide a table that illustrates the relation between types of equilibria and types of noise. In particular, there is a positive association between noise and positive equilibrium benefit for the decision maker.

[6] This fully-revealing equilibrium relies on a strong tie-breaking assumption that each persuader perfectly reveals the true state whenever he is indifferent. However, this is an unlikely prediction because all persuaders strictly prefer a less informative outcome. It is not sufficient to eliminate this fully-revealing equilibrium by focusing only on admissible equilibria because full revelation is not weakly dominated by any other strategy. For example, suppose that there are only two persuaders. Let $(x_1, y_1) \neq (0,1)$ be any arbitrary strategy from persuader 1 that is not fully revealing. Then, there exists some strategy $x_2 = 0$, $y_2 < 1$ from persuader 2 such that 1) persuader 2 always reports "negative" in state L, and 2) persuader 2 sometimes reports "negative" in state H with probability y_2. y_2 is a function of (x_1, y_1) and is chosen to be sufficiently low so that the decision maker chooses a_L when the result is "positive" from persuader 1 and "negative" from persuader 2. Given persuader 2's strategy, persuader 1 is strictly better off with the fully-revealing strategy (0, 1) than with (x_1, y_1). Therefore, the fully-revealing strategy is not weakly dominated.

To make the equilibrium analysis easier, I first introduce a notation that represents the decision maker's decision rule after she sees the test results.

Note that if a persuader chooses a test whose result is positive with the same probability in either state, then his test result is simply white noise. Call him *an uninformative persuader*. Since persuaders' test results are independent, adding or deleting uninformative persuaders has no impact on other players' equilibrium strategies. Therefore, the decision maker's action in equilibrium depends only on the strategy of informative persuaders.

Definition 2. A persuader i is *informative* if and only if $x_i < y_i$. Let N_I denote the set of all informative persuaders.

When persuaders' test designs are symmetric, the decision maker's decision rule can be characterized by a single number.

Definition 3. Suppose that all informative persuaders choose the same test (x,y). Then, $\alpha \in (0,1]$ is called the decision maker's *acceptance fraction* given (x,y) if her best response is to choose a_H if and only if the fraction of positive results from informative persuaders' tests is at least α.

Remark 1. In equilibrium, $\alpha \leq 1$ is well-defined because the decision maker must choose a_H if every test result is positive. Suppose that this is not the case and the decision maker never chooses a_H. In this case, any persuader has a profitable deviation to the most revealing test $(0,\bar{y})$, because the decision maker learns that the state is H and chooses a_H with certainty after observing a positive result from this test.

When persuaders' test designs are asymmetric, the analog of the acceptance fraction is an acceptance *set*: the decision maker chooses action a_H if and only if the observed positive results come from persuaders belonging to this set. This language is useful for analyzing asymmetric test profiles or deviations from symmetric test profiles.

Definition 4. Let the persuaders' tests be $(x_1,y_1),...,(x_n,y_n)$ and let $a \subseteq N_I$ denote the set of informative persuaders whose test results are positive. Then, $A \subseteq \mathscr{P}(N_I)$ is called the decision maker's *acceptance set* given $(x_1,y_1),...,(x_n,y_n)$ if her best response is to choose a_H if and only if $a \in A$.

For example, if there are two informative persuaders and the decision maker chooses a_H if and only if the test result from persuader 1 is positive, her acceptance set is $A =$

$\{\{1\},\{1,2\}\}$. If there are n informative persuaders and the decision maker chooses a_H if and only if all of their test results are positive, her acceptance set is $A = \{N_I\}$.

The argument in Remark 1 can be directly used to show that A is non-empty in any equilibrium. Moreover, since the decision maker is Bayesian and the test results are independent, an acceptance set must satisfy this: if $a_1 \in A$ and $a_1 \subset a_2$, then $a_2 \in A$. That is, more positive results cannot be less persuasive. The analog of a larger acceptance set in asymmetric equilibria is a lower acceptance fraction in symmetric equilibria.

Next, Proposition 1 provides a necessary and sufficient condition to identify whether the decision maker strictly benefits from persuaders' tests in an equilibrium. A non-beneficial equilibrium is identified by the smallest acceptance set (i.e., all test results must be positive in order to persuade the decision maker).

The key intuition behind the proof is simple yet important - the decision maker strictly benefits from a set of tests if and only if some realization of the test results can make her *strictly* prefer action a_H. When she has the smallest acceptance set in equilibrium, it implies that she never chooses a_H unless all test results are positive. Even in the latter case, she is merely indifferent between a_H and a_L, so her ex-ante expected utility is the same as when she chooses a_L without seeing any test. Proposition 1 is the backbone of all the other results in the paper. All formal proofs can be found in the Appendix.

Proposition 1. *In any equilibrium with $n \geq 1$, $p_d \in \left(\frac{1}{2}, 1\right)$ and $\bar{y} \in (0,1]$,*
$U = 0$ if and only if $A = \{N_I\}$.

Note that when there is only one persuader, he optimally designs his test so that a positive result successfully persuades the decision maker to choose a_H, i.e., $A = \{N_I\} = \{\{1\}\}$. Therefore, Proposition 1 immediately implies that when $n = 1$, the decision maker never benefits from the persuader's optimal test.

Corollary 1. *When $n = 1$, $U = 0$ for all $p_d \in \left(\frac{1}{2}, 1\right)$ and $\bar{y} \in (0,1]$.*

Moreover, if y is not bounded from above, an equilibrium is non-beneficial if and only if all persuaders' tests results are always positive in state H. When they are, any negative result perfectly reveals state L, so the decision maker's acceptance set must be $\{N_I\}$. When some persuader does not choose $y = 1$, he must be doing so in order to induce an acceptance set larger than $\{N_I\}$, which leaves a positive surplus for the decision maker. If y is bounded from above, all persuaders maximize y in all non-beneficial equilibria for the same reason, but they do so in some beneficial equilibria, too, as shown in the motivating example in Section 2.

Proposition 2. *Let $n \geq 1$ and $p_d \in \left(\frac{1}{2}, 1\right)$.*

(a) In any equilibrium with $\overline{y} = 1$, $U = 0$ if and only if $y_i = 1$ for all $i \in N_I$.

(b) When $\overline{y} < 1$, $y_i = \overline{y}$ for all i is a necessary but not sufficient condition for $U = 0$ in equilibrium.

	No exogenous noise: $\bar{y} = 1$	*Exogenous noise:* $\bar{y} < 1$
No endogenous noise: $y_i = \bar{y}$ for all i	Non-beneficial equilibria	?
Endogenous noise: $y_i < \bar{y}$ for some i	Beneficial equilibria	Beneficial equilibria

Table 1: Noise and types of equilibria

Table 1 summarizes Proposition 2's results. I say that there is *exogenous noise* in the game if $\bar{y} < 1$, and I say that a persuader i's test exhibits *endogenous noise* if $y_i < \bar{y}$. Proposition 2 shows that an equilibrium must be beneficial whenever some persuader's test exhibits endogenous noise.

Without exogenous noise ($\bar{y} = 1$), Proposition 3 shows that, while beneficial equilibria with endogenous noise always exist, non-beneficial equilibria always exist, too. Moreover, the persuaders always achieve their highest payoff in a non-beneficial equilibrium.

Proposition 3. *When $\bar{y} = 1$, for any $n \geq 2$ and $p_d \in \left(\frac{1}{2}, 1\right)$,*

(a) a symmetric equilibrium with $U = 0$ always exists. In this equilibrium, $y_i = 1$ for all $i \in N_I$. Moreover, the persuaders' ex-ante expected utility is maximized in this equilibrium;

(b) equilibria with $U > 0$ always exist, too. In these equilibria, $y_i < 1$ for some $i \in N_I$.

These findings suggest that, if one wishes to find an environment that guarantees information surplus for the decision maker, one should focus on environments with exogenous noise ($\bar{y} < 1$). In particular, if every equilibrium without endogenous noise is beneficial (i.e., the answer to "?" in Table 1 is "beneficial equilibria"), then every equilibrium is beneficial.

This motivates the next two subsections, which focuses on the case of $y_i = \bar{y} < 1$ for all i (the case marked with "?" in Table 1).

3.3 Symmetric binding tests and profitable deviation

This subsection lays the technical groundwork for analyzing symmetric equilibria with $y_i = \bar{y} < 1$ for all i. For simplicity, I call a test "binding" if $y_i = \bar{y} < 1$. I study cases in which all persuaders choose symmetric binding tests and I examine the properties of profitable deviations. Based on these results, the next subsection identifies which symmetric binding test profiles have profitable deviations and, in particular, when a non-beneficial test profile has a profitable deviation.

Definition 5. Let $\bar{y} \in (0,1)$. Let there be $n \geq 1$ persuaders and let k be an integer such that $1 \leq k \leq n$. A profile of tests is said to be *symmetric and binding with acceptance fraction* $\frac{k}{n}$ if every persuader chooses the same test design $(x^*, \bar{y})$ such that x^* satisfies the following equation:

$$\frac{\Pr(H|\text{exactly } k \text{ positive results})}{\Pr(L|\text{exactly } k \text{ positive results})} = \left(\frac{\bar{y}}{x^*}\right)^k \left(\frac{1-\bar{y}}{1-x^*}\right)^{n-k} = \frac{p_d}{1-p_d}. \tag{1}$$

Equation (1) implies that the decision maker is indifferent when exactly k out of n tests yield positive results. Therefore, this test profile induces the acceptance fraction $\frac{k}{n}$.

The following remark states that the symmetric and binding test profile with acceptance fraction $\frac{k}{n}$ is well-defined and unique. Its proof can be found in the Appendix.

Remark 2. For any $1 \leq k \leq n$, $p_d \in \left(\frac{1}{2}, 1\right)$, and $\bar{y} \in (0,1)$, there exists a unique x^* such that $x^* < \bar{y}$ and satisfies Equation (1). Moreover, x^* strictly increases with k.

The definition of symmetric and binding test profiles is more restrictive than its name suggests. It requires that the posterior likelihood given exactly k positive test results is exactly $\frac{p_d}{1-p_d}$. In principle, one can construct a symmetric test profile $(x, \bar{y})$ that satisfies two conditions: (1) the acceptance fraction induced by $(x, \bar{y})$ is $\frac{k}{n}$; (2) $\left(\frac{\bar{y}}{x}\right)^k \left(\frac{1-\bar{y}}{1-x}\right)^{n-k} > \frac{p_d}{1-p_d}$. However, this type of test profile never constitutes an equilibrium because any persuader has a profitable deviation to some higher x_i such that $\left(\frac{\bar{y}}{x_i}\right) \left(\frac{\bar{y}}{x}\right)^{k-1} \left(\frac{1-\bar{y}}{1-x}\right)^{n-k} = \frac{p_d}{1-p_d}$. Therefore, I focus only on tests that satisfy Equation (1).

When a symmetric and binding test profile has no profitable deviation, it is called a symmetric binding equilibrium.

Definition 6. A perfect Bayesian equilibrium is called a *symmetric binding equilibrium with acceptance fraction* $\frac{k}{n}$ if every persuader chooses the same test design $(x^*, \bar{y})$ such that x^* satisfies Equation (1).

In general, a persuader can deviate from a symmetric binding test profile in any number of ways; a deviation can induce the decision maker to expand, maintain, or shrink her acceptance set. Keeping track of profitable deviations may seem like a headache. However, Lemma 1 simplifies this process by eliciting the types of deviations that can be feasible and profitable.

Lemma 1. *Given $1 \leq k \leq n$, $p_d \in \left(\frac{1}{2}, 1\right)$, and $\bar{y} \in (0,1)$, suppose that all n persuaders start with the symmetric and binding test profile $(x^*, \bar{y})$ with acceptance fraction $\frac{k}{n}$. Following a unilateral deviation from persuader i to some arbitrary test design $\left(x^{'}, y^{'}\right) \neq (x^*, \bar{y})$, suppose that the minimum number of positive results required from the other non-deviating persuaders to induce a_H increases by*

c_1 when i's test result is negative, and

c_2 when i's test result is positive.

Then,

(a) $c_1 \geq 0$;

(b) if i's deviation to $\left(x^{'}, y^{'}\right)$ is profitable, $c_2 < 0$.

Part (a) of Lemma 1 states that a persuader can never make his negative result weak enough to replace a positive result. Therefore, if it originally takes k positive results to persuade the decision maker, now it must take at least k positive results to persuade her when the deviating persuader's result is negative.

Part (b) of Lemma 1 states that if a persuader wishes to profit from a deviation, he must reinforce his test so that his positive result can offset strictly more negative results from the other non-deviating persuaders.

Theorem 2 in the next subsection uses Lemma 1 directly to prove that the most beneficial symmetric binding equilibrium for the decision maker always exists. Lemmas 4 and 5 in the Appendix extend Lemma 1 to further characterize the functional form of the most profitable deviation. These two Lemmas are used to analytically identify cases when a non-beneficial equilibrium does not exist (Theorem 1) and to numerically perform welfare analysis for equilibria with a finite number of persuaders in the next subsection.

3.4 Decision maker's equilibrium benefits

As discussed at the beginning of Section 3, this subsection focuses on symmetric equilibria while the next one discusses equilibria in general. Here, I prove the first set of main results of the paper: in the case of $\bar{y} < 1$, when there are two or more persuaders, there does not exist any symmetric non-beneficial equilibrium if the number of persuaders n is sufficiently large or when the upper bound $\bar{y}$ is sufficiently low. In contrast, the decision maker's favorite beneficial symmetric binding equilibrium always exists. In the latter equilibrium, as n increases, each persuader strategically increases the informativeness of his individual test, making the decision maker strictly better off. As n goes to infinity, the decision maker learns the true state in every Pareto optimal symmetric equilibrium. These results show that the decision maker gains from additional persuaders and this gain is substantial.

The first step towards proving these results is to build a connection between the symmetric binding equilibria analyzed in the last section and symmetric equilibria in general. The following proposition shows that the set of symmetric non-beneficial equilibria contains a single element: the symmetric binding equilibrium with $\alpha = 1$. Any non-binding, non-beneficial symmetric test profile must have a profitable deviation towards the binding test. Therefore, the existence of symmetric non-beneficial equilibria is equivalent to the existence of the symmetric binding equilibrium with $\alpha = 1$.

Proposition 4. *A symmetric equilibrium is non-beneficial if and only if it is the symmetric binding equilibrium with* $\alpha = 1$.

The next Proposition shows that the decision maker's equilibrium payoff strictly decreases with α. Intuitively, the reason why a binding equilibrium has a lower acceptance fraction is because its tests are more informative and yield fewer false-positive results. Since test results are conditionally independent and the decision maker is Bayesian, the decision maker is better off with these more informative tests.

Proposition 5. *Given* $n \geq 2$*, suppose that there exist two symmetric binding equilibria with acceptance fractions* $\alpha_1 = \frac{k_1}{n}$ *and* $\alpha_2 = \frac{k_2}{n}$ *where* $k_1 < k_2$*. Let* U_1 *and* U_2 *denote the decision maker's ex-ante expected utility in these two equilibria. Then,* $U_1 > U_2$.

By Proposition 4 and 5, the symmetric binding equilibrium with $\alpha = 1$ is uniquely the worst symmetric equilibrium for the decision maker. By Proposition 5, the symmetric binding equilibrium with $\alpha = \frac{1}{n}$ (i.e., one positive result is sufficient to make the decision maker choose a_H) is uniquely the best symmetric binding equilibrium for the decision maker.

The next two theorems discuss the existence of these two equilibria when $n \geq 2$. Theorem 1 applies Lemma 4 to show that the worst symmetric equilibrium does not exist when

$\bar{y}$ is sufficiently low or when n is sufficiently large. In these cases, if we focus only on symmetric equilibria, the answer to "?" in Table 1 is "beneficial equilibria".

Theorem 1. *Non-existence of non-beneficial symmetric equilibria*

$\forall p_d \in \left(\frac{1}{2},1\right)$ *and* $n \geq 2$*, there exists some* $Y(n) > p_d$ *s.t.* Y *strictly increases in* n *and there does not exist any non-beneficial symmetric equilibrium if* $\bar{y} < Y(n)$*. When* $n \to \infty$*,* $Y(n) \to 1$.

$\forall p_d \in \left(\frac{1}{2},1\right)$ *and* $\bar{y} \in (0,1)$*, there exists some* $N(\bar{y}) \geq 2$ *s.t.* N *increases in* $\bar{y}$ *and there does not exist any non-beneficial symmetric equilibrium if* $n \geq N(\bar{y})$*. When* $\bar{y} \leq p_d$*,* $N(\bar{y}) = 2$.

When the exogenous noise or the number of persuaders is large, Theorem 1 shows that a persuader would rather deviate from a non-beneficial test to a more informative test whose positive result can offset negative results from the other persuaders. To see why, note that when the persuaders choose non-beneficial symmetric tests, the decision maker chooses a_H only when all test results are positive, which happens with probability $\frac{1}{2p_d}\bar{y}^n$. When $\bar{y}$ is low or n is high, this probability is low because it is difficult to avoid accidental negative results from a large group of independent persuaders. As the exogenous noise or the number of persuaders grows, even deviating to the most revealing test $(0,\bar{y})$ is rewarding: a positive result from this test can offset any number of negative results, and the decision maker chooses a_H with probability $\frac{1}{2}\bar{y}$ following this deviation, which is higher than $\frac{1}{2p_d}\bar{y}^n$ when $\bar{y}^{n-1}$ is lower than p_d.

The proof of Theorem 1 uses a more profitable deviation than $(0,\bar{y})$ to find tighter thresholds of $\bar{y}$ and n for the non-beneficial symmetric equilibrium to disappear. For all $n \leq 20$, I verified numerically that the deviation used in the proof is profitable for the widest range of $\bar{y}$ compared to other deviations. This allows me to identify the necessary and sufficient conditions for the non-beneficial symmetric equilibrium to disappear in the examples in Table 2. A little noise is enough to kill the non-beneficial test profile as an equilibrium.

	$p_d = 0.6$	$p_d = 0.8$
$n = 2$	$\bar{y} < 0.82$	$\bar{y} < 0.85$
$n = 3$	$\bar{y} < 0.90$	$\bar{y} < 0.92$
$n = 4$	$\bar{y} < 0.93$	$\bar{y} < 0.94$

Table 2: Necessary and sufficient condition under which the non-beneficial symmetric equilibrium does not exist (rounded to two decimal places)

If the non-beneficial symmetric tests often fail to constitute an equilibrium, which tests do? To get an idea, Figure 1 plots the decision maker's payoff in symmetric binding equilibria with different acceptance fractions in the cases of $n = 5$. If a value is missing for some α and $\bar{y}$, it means that the symmetric binding tests with acceptance fraction α do not constitute an equilibrium for that value of $\bar{y}$. Therefore, the domain of each curve in the graphs reveals the range of $\bar{y}$ over which a type of equilibrium exists.

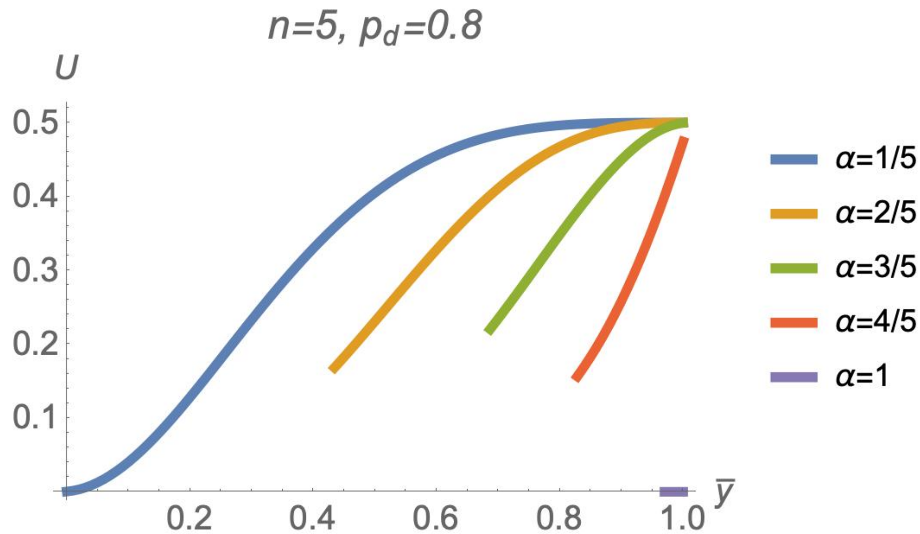

Figure 1: Decision maker's payoff in symmetric binding equilibria with different acceptance fractions

Two important observations arise from Figure 1: only the symmetric binding equilibrium with the lowest acceptance fraction $\alpha = \frac{1}{5}$ exists for the entire domain of $\bar{y}$ (the blue curve is the longest), and this equilibrium gives the highest payoff for the decision maker (the blue curve is the highest). Theorem 2 generalizes these two results.

Theorem 2. *When $n \geq 2$, for any $p_d \in \left(\frac{1}{2}, 1\right)$ and $\bar{y} \in (0,1)$, the symmetric binding equilibrium with acceptance fraction $\frac{1}{n}$ always exists. It is a beneficial equilibrium that induces the highest payoff for the decision maker among all symmetric binding equilibria.*

Theorem 2 uses Lemma 1 for the proof of existence. In the symmetric binding equilibrium with acceptance fraction $\frac{1}{n}$, if one persuader's result is positive, the decision maker chooses a_H even if she sees no positive result from the others. In other words, it is technically impossible to further decrease the number of positive results the decision maker requires from the other persuaders. Therefore, condition b in Lemma 1 is always violated and no deviation is profitable. The second half of Theorem 2 follows directly from Proposition 5, which states that the symmetric binding equilibrium with the lowest acceptance fraction gives the highest payoff to the decision maker.

The existence results in Theorem 1 and 2 established that, as the number of persuaders increases, the decision maker's expected gain is positive in any symmetric equilibrium. The rest of the subsection discusses the magnitude of this positive gain. How large is it? Given that the persuaders can endogenously adjust their tests as their numbers grow, do they simply keep the decision maker's gain at a positive but negligible amount? How much does the decision maker learn about the true state when she receives information from infinitely many persuaders?

Proposition 6 and Theorem 3 provide favorable answers for the decision maker. Her gain grows with the number of persuaders and is far from being negligible. She practically learns the true state in any Pareto optimal symmetric equilibrium when there are infinitely many persuaders.

Proposition 6. *Given $\bar{y} \in (0,1)$, for each n, let $U^*(n)$ denote the decision maker's expected utility in her favorite symmetric binding equilibrium. Let $(x_n^*, \bar{y})$ denote each persuader's test in this equilibrium. Then, as n increases, $U^*(n)$ strictly increases and the false-positive probability x_n^* strictly decreases.*

One might think that $U^*(n)$ increases with n simply because the decision maker sees more tests. While this is true, Proposition 6's result is more powerful: as the number of persuaders increases, each persuader also strategically increases the informativeness of his individual test. Intuitively, the decision maker earns the most in a symmetric binding equilibrium that has the most informative test design. This is the design that has the lowest false-positive probability so that a single positive result offsets all of the rest $n-1$ negative results. This implies that, as n increases, a positive result must be able to offset a larger number of negative results, and this is achieved only through a further decrease in the false-positive probability. Therefore, as n increases, the decision maker not only learns more test results, but each test result also reveals more information about the true state. This is the reason why the decision maker's maximum expected utility strictly increases with n.

Figure 2 illustrates the rate of increase in $U^*(n)$ as n increases. Observe that $U^*(n)$ quickly converges to $\bar{U} = 0.5$, the decision maker's utility when she learns the true state. This is no coincidence. In fact, as long as $\bar{y} < 1$, the decision maker asymptotically learns the true state in all Pareto optimal symmetric equilibria.

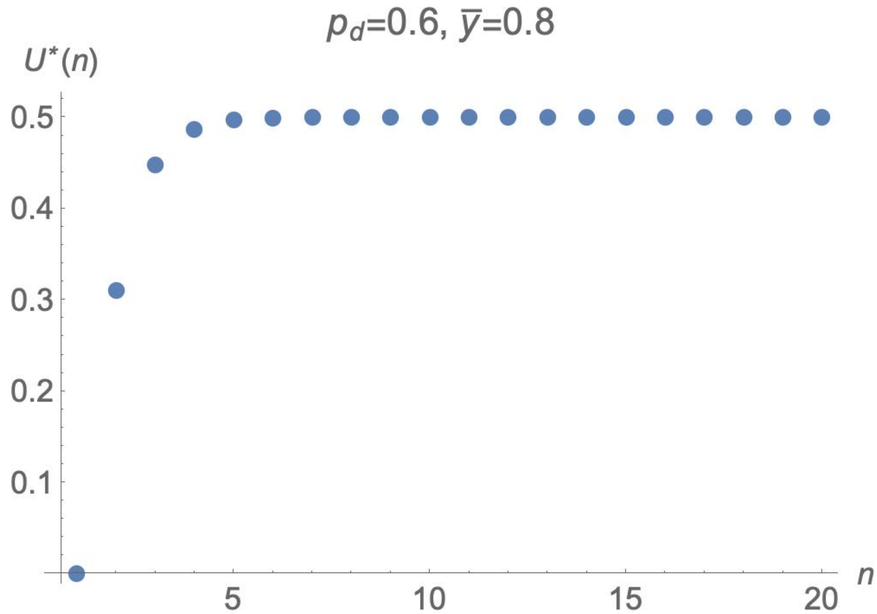

Figure 2: The decision maker's maximum expected utility in a symmetric binding equilibrium increases and converges to $\bar{U} = 0.5$, her utility when she learns the state.

Theorem 3. *For all $\bar{y} < 1$, as $n \to \infty$, the decision maker learns the true state with probability 1 in all Pareto optimal symmetric equilibria.*

Theorem 3 is a substantial step-up from Proposition 6 because the decision maker's large asymptotic information gain no longer relies on the selection of her favorite equilibrium. The persuaders fully reveal the true state even in the persuader-optimal symmetric equilibrium.

Intuitively, because of the exogenous noise, the persuaders asymptotically opt for informative test designs with relatively low acceptance fractions because it is increasingly difficult to avoid negative results as the number of tests expands. In these informative tests, the false-positive probability x is strictly lower than the true-positive probability y. By the law of large numbers, the decision maker can distinguish state H from state L by simply observing whether the actual fraction of positive results is x or y. In contrast, recall from Proposition 3 that when $\bar{y} = 1$, there is always a symmetric binding equilibrium in which the acceptance fraction is 1 and the decision maker's expected utility is 0. In this case, the persuaders make their tests less informative as their number increases, to the extent that the fraction of positive results converges to 1 asymptotically regardless of the state.

The proof of Theorem 3 shows that the symmetric binding equilibrium with acceptance fraction $\alpha = \frac{1}{n}$ belongs to the group of Pareto optimal symmetric equilibria. Therefore, the decision maker's expected utility in this equilibrium converges to the truth-learning utility $\bar{U}$ as $n \to \infty$, as illustrated in Figure 2.

To summarize, Proposition 6 and Theorem 3 show that the decision maker benefits substantially from multiple persuaders. If she can always consult additional persuaders at no cost (which is the assumption of this paper), she has a strong incentive to seek as many persuaders as possible. Only when seeking additional persuaders is costly will she seek only finitely many persuaders, because her information gain is bounded at $\bar{U}$.

3.5 Persuaders' equilibrium benefits

When exogenous noise is present and persuaders play a symmetric equilibrium, the previous subsection shows that the decision maker benefits from having many persuaders. As discussed at the beginning of Section 3, there are realistic reasons to believe that identical, independent persuaders are likely to play a symmetric equilibrium. Nevertheless, should the persuaders have the ability to coordinate on an asymmetric test profile, key intuitions and results from the analysis of symmetric equilibria can extend to asymmetric equilibria, as well.

This subsection looks at all symmetric and asymmetric equilibria and discusses how the persuaders' welfare changes with the number of persuaders. Compared to the case of only one persuader, is their expected utility higher or lower when there are many of them? If the persuaders can coordinate to play their most preferred equilibrium that is potentially asymmetric, does the decision maker receive any benefit from their tests?

I show that when there is sufficient exogenous noise, the persuaders' expected utility is actually higher in a beneficial equilibrium with multiple informative persuaders than in the non-beneficial equilibrium with only one persuader. This shows that increasing the number of persuaders is a Pareto improvement that increases the payoff of *both* the decision maker and the persuaders. This result is also used to show that the persuader-optimal equilibrium, symmetric or not, must strictly benefit the decision maker.

To prove these results, I first define the persuader's payoff when there is only one persuader as a bench mark. Recall that in this equilibrium, the persuader chooses the test $(x, y) = \left(\frac{1-p_d}{p_d}\bar{y}, \bar{y}\right)$ to make the decision maker indifferent when she sees a positive result.

The decision maker's expected utility is 0 and the persuader's expected utility is

$$
\begin{aligned}
V_{solo} &= \frac{1}{2}\left(\frac{1-p_d}{p_d}\bar{y}+\bar{y}\right) \\
&= \frac{\bar{y}}{2p_d}.
\end{aligned}
$$

Definition 7. $V_{solo} = \frac{\bar{y}}{2p_d}$ is the persuader's ex-ante expected utility in the equilibrium of the game with only one persuader.

The rest of this subsection compares V_{solo} with the persuaders' expected utility in the decision maker's favorite symmetric binding equilibrium with multiple persuaders. Theorem 4 shows that when $\bar{y}$ is sufficiently low, the latter is strictly higher than V_{solo}. Everyone in the game is better off with multiple informative persuaders.

Theorem 4. *When $n \geq 2$, let V_n denote each persuader's payoff in the beneficial symmetric binding equilibrium with $\alpha = \frac{1}{n}$. For any $p_d \in \left(\frac{1}{2}, 1\right)$, there exists a unique $\bar{y}^* \in (p_d, 1)$ such that $V_n > V_{solo}$ if and only if $\bar{y} < \bar{y}^*$.*

Intuitively, in the equilibrium with one persuader, the decision maker never chooses a_H whenever the test result is negative. In the symmetric equilibrium with $n \geq 2$ persuaders and an acceptance fraction of $\frac{1}{n}$, even if many test results are negative, the decision maker still chooses a_H as long as some test result is positive. When there is sufficient noise, the probability of negative results is large regardless of the test design. Therefore, a persuader benefits from the low acceptance fraction in the n-persuader equilibrium.

To illustrate how low $\bar{y}$ must be in order for a persuader to prefer the beneficial n-persuader equilibrium over the non-beneficial one-persuader equilibrium, Figure 3 plots the value of the threshold $\bar{y}^*$ (thick red line) as a function of p_d when $n = 2$. Note that $\bar{y}^*$ is above the 45-degree line because it is always higher than p_d for all $n \geq 2$. Also note that $\bar{y}^*$ is lower than the value $Y(2)$ from Theorem 1, which means that when the condition $\bar{y} < \bar{y}^*$ is satisfied, the two-persuader symmetric binding equilibrium with $\alpha = \frac{1}{2}$ is the only two-persuader symmetric binding equilibrium. Numerically, for all $n \leq 20$, $\bar{y}^*$ is always below $Y(n)$. Moreover, $\bar{y}^*$ is strictly decreasing in n and converging to p_d as $n \to \infty$. The next Theorem formally proves the asymptotic convergence of $\bar{y}^*$ to p_d.

Theorem 5. *When $\bar{y} < 1$, as $n \to \infty$, each persuader's payoff converges to $V_\infty = \frac{1}{2}$ in every Pareto optimal symmetric equilibrium because the decision maker learns the true state with probability 1. $V_\infty > V_{solo}$ if and only if $\bar{y} < p_d$.*

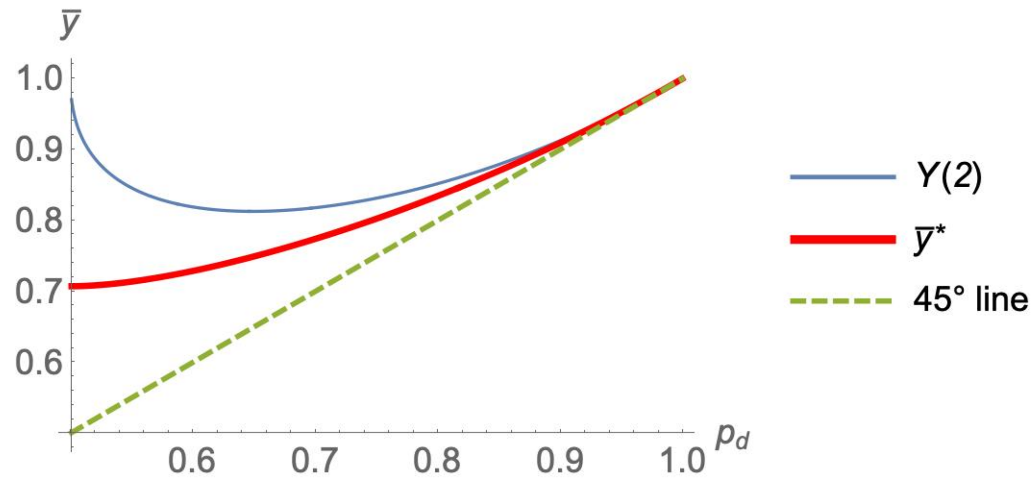

Figure 3: When $\bar{y} < \bar{y}^*$, a persuader's payoff is higher in the beneficial two-persuader symmetric binding equilibrium than in the one-persuader equilibrium. $\bar{y}^*$ is higher than p_d and lower than $Y(2)$, the threshold below which no two-persuader non-beneficial symmetric equilibrium exists.

Theorem 4 and 5 imply that, when there is sufficient noise, the persuaders' favorite equilibrium is not the asymmetric equilibrium in which only one persuader is informative. The rest of this section investigates this implication and uses it to characterize the persuader-optimal equilibrium.

I first show that the asymmetric equilibrium with only one informative persuader is a persuader's most-preferred non-beneficial equilibrium. Therefore, if a persuader prefers a beneficial equilibrium over this asymmetric equilibrium, his favorite equilibrium must induce positive benefit for the decision maker. Theorem 6 formalizes this and proves that when there is sufficient exogenous noise, a persuader's most preferred (asymmetric or symmetric) equilibrium must be beneficial for the decision maker.

To start with, Lemma 2 proves that a single persuader can replicate the outcome of any n-persuader, non-beneficial test profile. He achieves this by setting his probability of positive test result equal to the probability of having n positive results in the n-persuader test profile in either state.

Lemma 2. *For any $n \geq 1$, suppose that when n persuaders choose some arbitrary non-beneficial test profile, each persuader's ex-ante expected utility is V. Then, in the game with only one persuader, this persuader has a feasible test that also gives him an expected utility of V. Therefore, V_{solo} is the maximum ex-ante expected utility that any persuader can achieve in any non-beneficial test profile regardless of n.*

Because V_{solo} is the upper bound of a persuader's payoff in any non-beneficial equilibrium, if there is an equilibrium in which only one persuader is informative, this must be the best non-beneficial equilibrium for the persuaders. Lemma 3 confirms that this asymmetric equilibrium always exists.

Lemma 3. *For any $n \geq 1$, there always exists an asymmetric equilibrium with only one informative persuader and $n-1$ uninformative persuaders, and it is the best non-beneficial equilibrium for the persuaders.*

Lemma 3 implies that if there exists an *n*-persuader, beneficial equilibrium such that the persuaders prefer this equilibrium over being alone, then they must prefer this equilibrium over any *n*-persuader, non-beneficial equilibrium. This, in turn, implies that the persuaders' favorite equilibrium must be a beneficial one for the decision maker. Based on Theorems 4 and 5, this is the case when there is sufficient exogenous noise.

Theorem 6. *When $n \geq 2$, for any $p_d \in \left(\frac{1}{2}, 1\right)$, there exists some $\bar{y}^* \in (p_d, 1)$ such that the best equilibrium for the persuader is beneficial for the decision maker if $\bar{y} < \bar{y}^*$;*

When $n \to \infty$, the best equilibrium for the persuader is beneficial for the decision maker if $\bar{y} < p_d$.

Theorem 6 shows that even if the persuaders can choose to play their favorite equilibrium, they will endogenously leave information surplus to the decision maker as long as sufficient exogenous noise is present. Remark 3 below shows that it is difficult to give a general further characterization of the persuader-optimal equilibrium. Therefore, Theorem 6 is particularly useful because it shows that persuader-optimal equilibrium is beneficial without explicitly calculating this equilibrium. Moreover, Theorem 6 also implies that the decision maker can benefit when one persuader is in charge of multiple tests, or when the persuaders choose test designs sequentially. These cases are discussed in more details in Section 4.

Remark 3. Persuader-optimal equilibrium

In general, it is difficult to characterize the persuader-optimal equilibrium outcome analytically because the associated tests are not necessarily symmetric or binding. For example, suppose that $n = 2$ and $\bar{y} = 0.7$. For all $p_d \geq 0.7$, the persuader-optimal equilibrium is the beneficial symmetric binding equilibrium with acceptance fraction $\frac{1}{2}$. However, when $p_d = 0.6$, the persuader-optimal equilibrium is asymmetric: one persuader chooses a binding test $(x_1, y_2) = (0.2417, 0.7)$ and the other chooses a non-binding test

$(x_2, y_2) = (0.1472, 0.5583)$. Nonetheless, the decision maker benefits in this equilibrium with $U = 0.3374 > 0$.

The non-tractability of persuader-optimal equilibrium highlights the usefulness of Theorem 6: when $\bar{y}$ is sufficiently low, one can tell that the persuader-optimal equilibrium is beneficial for the decision maker without even knowing what this equilibrium looks like. For example, as illustrated in Figure 3, $\bar{y}^* > 0.7$ for all values of p_d when $n = 2$. Therefore, in the example of $\bar{y} = 0.7$, Theorem 6 implies that the persuader-optimal equilibrium is always beneficial for the decision maker regardless of p_d.

To conclude, results in sections 3.4 and 3.5 send a unified message: when there is sufficient noise that prevents a persuader from perfectly identifying his favorable state, introducing multiple persuaders strictly benefits *both* the decision maker and the persuaders. The persuaders choose a more informative test profile in equilibrium when there are many of them, and this extra information improves the decision maker's welfare. The persuaders benefit from this act themselves because, in response to their informative test designs, the decision maker is willing to choose the persuaders' preferred action upon seeing relatively few positive test results.

4 Discussion of alternative modeling choices

In this section, I discuss the robustness of the main results with respect to alternative modeling choices.

A. Single persuader with multiple tests

This paper focuses on games with multiple identical persuaders, each of whom independently conducts one test. One might ask how the equilibrium outcomes change when there is only one persuader in charge of multiple independent tests.

If this persuader can choose asymmetric tests, then the equilibrium is outcome-equivalent to the persuader-optimal equilibrium of the multi-persuader game. As discussed in Section 3.5, the equilibrium outcome is beneficial for the decision maker when $\bar{y}$ is sufficiently low. When $\bar{y}$ is high, the monopolist persuader may opt for tests that no longer benefits the decision maker. In contrast, in the multi-persuader game, beneficial equilibrium for the decision maker always exists. In this sense, the decision maker does not prefer to delegate all tests to a monopolist persuader.

If the monopolist persuader must make all tests identical, then one can analytically compare his optimal tests with the symmetric tests studied in Section 3.4. Whether the decision maker benefits more in a multi-persuader game or a monopolist-persuader game is ambiguous. In some cases, the monopolist prefers non-beneficial tests even when they do not constitute an equilibrium in the multi-persuader game. In other cases, the monopolist does not prefer non-beneficial tests but they constitute an equilibrium in the multi-persuader game. To understand why, note that the monopolist compares the non-beneficial tests with deviations to other *symmetric* test profiles. In comparison, in the equilibrium analysis of the multi-persuader game, one considers only *asymmetric* deviations in which only one test is unilaterally changed. Which is the more profitable type of deviation is ambiguous and depends on other parameters of the game. This is the reason why replacing multiple identical persuaders with a monopolist who must choose identical tests can increase or decrease a decision maker's payoff. Nevertheless, Theorem 6 still applies to predict the monopolist's choice between beneficial and non-beneficial tests: when $n > 1$ and there is sufficient exogenous noise that prevents the monopolist to perfectly identify his favorable state, he prefers beneficial tests over non-beneficial tests.

B. Correlated test results

In this paper, I assume that the persuaders' test results are conditionally independent. The Introduction mentioned that Gentzkow and Kamenica's studies (2017a, 2017b) applied to the opposite case, in which persuaders' test results are arbitrarily correlated. When identical persuaders' test results are conditionally independent, beneficial equilibria always exist and can be the only equilibria; when persuaders' test results are arbitrarily correlated, only non-beneficial equilibria exist. In many scenarios, it is naturally appropriate to assume that test results are conditionally independent, especially when the persuaders do not belong in the same organization. Nevertheless, one may ask what happens when test results are partially correlated. Here, I argue that equilibrium outcomes with partial correlation are more likely to resemble the outcomes in this paper.

Borrowing from the modeling language in Li and Norman (2018), say that the result of a test depends on the true state and the realization of a sunspot variable independent of the state (e.g., a random variable uniformly distributed on $[0,1]$). The sunspot variable is responsible for the randomness of the test result. Then, test results from n persuaders are *arbitrarily correlated* if there is a public sunspot variable and all n test results are conditioned on it; test results from persuaders are *independent* if there are n i.i.d. private sunspot variables, and each persuaders' test result is conditioned on his private sunspot. One way to model partial correlation is to say that test results from n persuaders are *partially correlated* if they are conditioned on the public sunspot with probability z and the private sunspots with probability $1-z$ for some $z \in (0,1)$, assuming that all players, including the decision maker, know which sunspot(s) is(/are) actually used. It does not matter if the persuaders are restricted to designing tests independent of the sunspot selection or if their test designs can be conditional on the sunspot selection. In either case, it is with positive probability that persuader i's test result will be different from j's. In particular, the number of positive results in state L will be random, and this randomness incentivizes the persuaders to expand the decision maker's acceptance set by designing more-revealing tests with informative positive results. This gives rise to equilibria that benefit the decision maker.

C. Sequential persuaders

In this paper, persuaders choose their tests simultaneously. Does the decision maker have any incentive to approach the persuaders sequentially?

The answer is "no" if later persuaders can observe both the test designs and test results of the previous persuaders. The ability to observe previous test results allows the persuaders to better coordinate on their tests and extract all surplus from the decision maker. In the subgame-perfect equilibrium, if all previous test results are negative, the next persuader chooses a test whose positive result increases the decision maker's posterior belief to exactly p_d (consequently, the decision maker chooses a_H). If some previous test result is positive, all later persuaders choose uninformative tests and the decision maker's posterior belief stays at p_d. The decision maker's ex-ante expected utility is $U = 0$ because she never strictly prefers to switch to a_H. As the number of persuaders increases to infinity, the persuaders' ex-ante expected utility converges to their first-best payoff even when exogenous noise exists. Therefore, revealing previous persuaders' test designs and results to later persuaders benefits the persuaders but not the decision maker. Proof of this result can be found in Section 6.20 of the Appendix.

If later persuaders can observe the test designs but *not* the test results, the subgame-perfect equilibrium is outcome-equivalent to the equilibrium of a game in which one persuader dictates all n tests (Section 4.A), which is outcome-equivalent to the persuader-optimal equilibrium in the simultaneous game (Section 3.5). In other words, the subgame-perfect equilibrium outcome of this particular sequential game is an element of the set of equilibrium outcomes in the simultaneous game, and the latter set can contain other equilibrium outcomes that are better or worse for the decision maker. Therefore, it is difficult to answer definitely whether the decision maker prefers to reveal test designs of previous persuaders to later persuaders. What one can say is that the decision maker's expected payoff in this sequential game is positive when $\bar{y}$ is sufficiently low, as implied by Theorem 6 in section 3.5.

D. Exogenous bounds on the false-positive probability

In this paper, I assume that the false-positive probability, x, is unbounded from below. This assumption is made for three reasons. First, because the persuaders already choose substantial false-positive probabilities endogenously in every equilibrium, a small lower bound on x is usually ineffective. (In contrast, the persuaders maximize y in many equilibria, so an upper bound on y is indeed effective.) Second, Proposition 2 shows that the value of y, not x, identifies non-beneficial equilibria. Therefore, it is more important to focus on the effect of a bound on y rather than that of one on x. Third, when only the upper bound on y is imposed, the persuaders still have the freedom to endogenously choose from a rich set of test designs that can induce any possible acceptance fraction. When a binding lower bound on x is introduced in addition to the upper bound on y, the equilibrium outcomes are directly shaped by the exogenous values of these bounds and the set of feasible acceptance fractions shrinks exogenously. These scenarios are relatively uninteresting as they deviate from the focus of the information design literature, which has largely been on the *endogenous* information outcomes.

Examples in Section 6.21 of the Appendix illustrates the effect of a bound on x. In general, the effect of a lower bound $\underline{x}$ on the set of equilibria is ambiguous. On the one hand, it can kill equilibria with low acceptance fractions. On the other hand, it can prevent deviations to tests with a low false-positive probability, thus creating new equilibria. In contrast, an upper bound on x prevents high false-positive probabilities in the test designs. Therefore, it eliminates equilibria with poorly informative tests and small acceptance sets, such as the non-beneficial equilibria.

E. Continuous state space

This paper does not hinge on the assumption of the binary state space. Suppose that the true state is a continuous variable $z \in \mathbb{R}$. If the action space of the decision maker is still

$\{a_H, a_L\}$ and the persuaders still strictly prefer a_H regardless of the true state, then the persuaders adopt a threshold strategy (Kolotilin, 2015) and the main results of the paper still apply.

For example, when there are two persuaders, there exists a symmetric equilibrium in which both persuaders choose tests that yield positive results when $z \geq \bar{z}$ and negative results when $z < \bar{z}$, where $\bar{z}$ is a relatively high[7] threshold chosen in such a way that the decision maker is indifferent when exactly one test result is positive. Since the acceptance fraction is less than one, the decision maker strictly benefits from the tests.

F. Non-binary test results

Recall that a test generates a message $m \in M$. In this paper, $M = \{positive, \ negative\}$. In general, this binary assumption is not without loss of generality when there are multiple persuaders, but relaxing this assumption only strengthens this paper's results. Specifically, relaxing the binary restriction on M does not change the set of non-beneficial equilibrium outcomes with $U = 0$; it only increases the number of beneficial equilibrium outcomes with $U > 0$. Since beneficial equilibria with $U > 0$ already exist when M is binary, they continue to exist when M is larger; if non-beneficial equilibria do not exist when M is binary, they still do not exist when M is larger; if persuaders prefer some beneficial equilibrium over a non-beneficial equilibrium when M is binary, they continue to exhibit this preference when M is larger. Hence, the results of this paper are robust when the binary restriction of M is relaxed. Section 6.21 in the Appendix proves this result.

G. Non-uniform prior belief

The assumption that the prior probabilities of the states, $\Pr(H)$ and $\Pr(L)$, are both equal to $\frac{1}{2}$ is without loss of generality. When $\Pr(H) \neq \Pr(L)$, all analyses in this paper applies as long as $p_d > \Pr(H)$, so that the decision maker's default action is a_L. To be specific, the prior belief influences the game dynamics only through the decision maker's decision rule: she chooses a_H if and only if

$$\frac{\Pr(H \mid \text{test results})}{\Pr(L \mid \text{test results})} \geq \frac{p_d}{1 - p_d}.$$

[7] compared to the threshold in the game with only one persuader.

By the Bayes' rule, this is equivalent to

$$\frac{\Pr(\text{test results} \mid H)}{\Pr(\text{test results} \mid L)} \geq \frac{p_d}{1-p_d} \cdot \frac{\Pr(L)}{\Pr(H)}$$

The persuaders' strategies best respond to the last inequality. In this paper, because $\Pr(H) = \Pr(L) = \frac{1}{2}$, the right-hand side of the inequality is simply $\frac{p_d}{1-p_d}$. When $\Pr(H) \neq \Pr(L)$, the right-hand side of the inequality becomes $\frac{p_d}{1-p_d} \cdot \frac{\Pr(L)}{\Pr(H)}$, which is still an exogenous constant number. Therefore, the case of $p_d > \Pr(H) > \frac{1}{2}$ is outcome-equivalent to a case with $\Pr(H) = \frac{1}{2}$ and a lower threshold of doubt $p_d^{'} < p_d$; the case of $\Pr(H) < \frac{1}{2}$ and $p_d > \Pr(H)$ is outcome-equivalent to a case with $\Pr(H) = \frac{1}{2}$ and some higher threshold of doubt $p_d^{'} > p_d$.

5 Conclusion

This paper studies a game in which a decision maker obtains information about a payoff-relevant state only through persuaders of the same type: all persuaders want the decision maker to choose a particular action regardless of the state, and they promote this action by independently designing truthful but biased tests on the state to influence the decision maker's belief.

While the decision maker never benefits in an equilibrium with only one persuader, this paper shows that if she allows two or more persuaders to test the true state for her, she can strictly benefit from their information even though the persuaders are all identical. If some exogenous noise prevents the persuaders from perfectly identifying their favorable state and the number of persuaders is sufficiently large, the decision maker benefits in every symmetric equilibrium. As the number of persuaders goes to infinity, they reveal the true state in every Pareto optimal symmetric equilibrium. Moreover, when there is sufficient exogenous noise, a persuader also benefits from the presence of other persuaders, so it is a Pareto improvement to increase the number of persuaders from one to many.

These results uncover a novel insight that explains from a new angle why more persuaders reveal more information. They do it because of a cooperative motive: by lowering the false-positive probability in his test design, a persuader makes a favorable result from his test convincing enough to offset potential unfavorable results from other fellow persuaders. As a result of fewer false-positive test results, the decision maker benefits more from the persuaders' information.

The game studied in this paper has many applications (e.g., a government learns from lobbyists; a consumer learns from sellers). This paper provides an easy solution to improve a decision maker's welfare. If the decision maker does not have the expertise to test the true state herself, nor does she have the budget to hire unbiased third-party experts, she can still effectively learn about the true state by simply permitting more than one biased persuader to test the true state for her.

6 Appendix

6.1 Proof of Proposition 1

The "only if" part is proved by contraposition. The same argument in Remark 1 can be directly used to show that $A \neq \emptyset$ in any equilibrium. Define $\{\mu_i\}$ and $\{\mu_j\}$ such that, in equilibrium, the revealed test results induce posterior belief $\Pr(H) = \mu_i < p_d$ with probability q_i and $\Pr(H) = \mu_j \geq p_d$ with probability q_j. For μ and q to be well-defined, they must satisfy two conditions: (1) $\sum_i q_i + \sum_j q_j = 1$ and (2) $\sum_i q_i\mu_i + \sum_j q_j\mu_j = \frac{1}{2}$ (the expectation of posterior belief is equal to the prior belief). Now, suppose that $A \neq \{N_I\}$. The decision maker weakly prefers a_H even when some test result is negative. Hence, she must strictly prefer a_H when all test results are positive. This implies that $\mu_j > p_d$ for some j and $\sum_j q_j \left(\mu_j - p_d\right) > 0$. The decision maker's ex-ante expected utility is

$$
\begin{aligned}
U &= \sum_i q_i \cdot 0 + \sum_j q_j \left[\mu_j - \left(1-\mu_j\right)\frac{p_d}{1-p_d}\right] \\
&= \frac{1}{1-p_d} \cdot \sum_j q_j \left(\mu_j - p_d\right) \\
&> 0
\end{aligned}
$$

This proves that $U > 0$ when $A \neq \{N_I\}$.

For the "if" part, note that $A = \{N_I\}$ means that the decision maker chooses a_H if and only if all tests results are positive. It also implies that, when all results are indeed positive, the decision maker must be precisely indifferent between a_H and a_L. To see why, first suppose that the decision maker strictly prefers a_H when all results are positive. This induces a profitable deviation for persuader 1: he can strictly increase his payoff by increasing x_1 until the decision maker becomes indifferent when all results are positive. This deviation increases the probability of a_H without changing the acceptance set. Next, suppose that the decision maker strictly prefers a_L when all results are positive. This implies that she never chooses a_H, which contradicts the assumption that $A = \{N_I\}$. Therefore, in any equilibrium with $A = \{N_I\}$, the decision maker must be indifferent when all test results are positive. In other words, either she chooses a_L when some test fails, or she is indifferent between a_H and a_L when no test fails. As a result, the decision maker's ex-ante expected utility is equal to her utility when she always chooses a_L unconditionally - i.e., $U = 0$.

6.2 Proof of Corollary 1

When there is only one persuader, he optimally designs the test so that the decision maker chooses a_H if and only if the test result is positive. Therefore, $A = \{N_I\} = \{\{1\}\}$, which implies that $U = 0$ by Proposition 1.

6.3 Proof of Proposition 2

Proposition 1 shows that proving $U = 0$ is equivalent to proving $A = \{N_I\}$. Therefore, it is sufficient to prove the case of $\bar{y} = 1$ by showing that $A = \{N_I\}$ if and only if $y_i = 1$ for all $i \in N_I$.

To prove the "only if" part, suppose that $A = \{N_I\}$ but $y_i < 1$ for some i. This cannot be an equilibrium because persuader i strictly benefits from an increase in y_i. All else equal, this upward deviation of y_i increases the probability of yielding a positive result. It also increases the induced posterior belief when the result is positive, which implies that the acceptance set does not shrink in response to the deviation. Therefore, such a deviation strictly increases the probability of a_H, and persuader i strictly prefers to deviate to $y_i = 1$.

To prove the "if" part, note that if informative tests never yield negative results in state H, a negative result from a single informative test perfectly reveals state L. Therefore, the decision maker never chooses a_H when seeing an informative negative result - i.e., $A = \{N_I\}$.

When $\bar{y} < 1$, one can show that $A = \{N_I\} \Rightarrow y_i = \bar{y}$ for all i by replicating the previous proof for the "only if" part in the case of $\bar{y} = 1$. By Proposition 1, this implies that $y_i = \bar{y}$ for all i is a necessary condition for $U = 0$ in equilibrium. The motivating example in Section 2 shows that there generally exist equilibria in which $y_i = \bar{y}$ for all i, but $U > 0$.

6.4 Proof of Proposition 3

Proof of (a): By Kamenica and Gentzkow (2011), if there is a single persuader who can design any test with a finite result space, his maximized ex-ante expected utility is $V^* = \frac{1}{2p_d}$. For example, he can achieve V^* by designing a test whose result is either positive or negative with $\Pr(positive \mid H) = 1$ and $\Pr(positive|L) = \frac{1-p_d}{p_d}$ (given these probabilities, the decision maker is indifferent when the test result is positive). In the multi-persuader game studied in this paper, V^* is the upper-bound of persuaders' feasible ex-ante expected utility. To see why, note that if persuaders in this paper choose tests $\{(x_i, y_i) \mid i = 1, 2, ..., n\}$, the single persuader in Kamenica and Gentzkow's setting can always design a feasible finite-result test that is outcome-equivalent to $\{(x_i, y_i) \mid i = 1, 2, ..., n\}$. Therefore, the persuaders' payoff in this paper's setting can never exceed the maximized payoff of the single persuader in Kamenica and Gentzkow's setting. Next, I propose a non-beneficial symmetric test profile that achieves exactly V^* and, therefore, constitutes an equilibrium with the highest payoff for the persuaders.

Consider the following symmetric test profile: for all persuaders $i = 1, 2, ..., n$,

$$x_i = x \equiv \left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}, \; y_i = y \equiv 1.$$

The decision maker's best response is to choose a_H if and only if all persuaders' test results are positive, in which case her posterior belief is p_d. The persuaders' ex-ante expected utility is

$$\begin{aligned} V &= \frac{1}{2}(y^n + x^n) \\ &= \frac{1}{2}\left(1 + \frac{1-p_d}{p_d}\right) \\ &= \frac{1}{2p_d} \\ &= V^*. \end{aligned}$$

Since V^* is the upper bound of the persuaders' feasible payoff, any unilateral deviation to some $\left(x', y'\right) \neq (x, y)$ will weakly decrease the deviator's payoff. Therefore, the proposed test profile constitutes a symmetric equilibrium that maximizes the persuaders' payoff. By Proposition 2, because $y = 1$, this test profile induces $U = 0$.

Proof of (b): Proposition 2 shows that an equilibrium yields $U > 0$ if and only if some informative persuader i chooses $y_i < 1$. Here, I prove that this type of equilibrium always exists for any $p_d \in \left(\frac{1}{2}, 1\right)$ and any $n \geq 2$.

Claim 1: A symmetric equilibrium with $U > 0$ always exists when $n = 2$.

Proof of Claim 1: Suppose that both persuaders choose the same test (x, y) such that

$$0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}} \text{ and } y = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)} < 1.$$

The upper bound of x makes sure that y is well-defined and $y \in (x, 1)$..

Note that if x and y satisfy these conditions,

$$\frac{y}{x} \cdot \frac{1-y}{1-x} = \frac{p_d}{1-p_d}.$$

Responding to (x, y), the decision maker's acceptance set is $A = \{\{1\}, \{2\}, \{1,2\}\}$ (i.e., chooses a_H upon seeing at least one positive result). Each persuader's ex-ante expected utility is

$$\begin{aligned} V &\equiv \frac{1}{2}\left[2y(1-y) + y^2 + 2x(1-x) + x^2\right] \\ &= \frac{1}{2}\left(2y - y^2 + 2x - x^2\right). \end{aligned}$$

I now go through each case of unilateral deviation to verify that the proposed tests form an equilibrium.

(1) Suppose that persuader 1 deviates to some (x_1, y_1) s.t. the decision maker's acceptance set is still $A = \{\{1\}, \{2\}, \{1,2\}\}$. To induce A, x_1 and y_1 must satisfy

$$\begin{aligned} \frac{y_1}{x_1} \cdot \frac{1-y}{1-x} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}, \\ \frac{y}{x} \cdot \frac{1-y_1}{1-x_1} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}. \end{aligned}$$

The first inequality implies that

$$x_1 \leq \frac{y_1}{y} \cdot x,$$

and the second inequality implies that

$$\begin{aligned}
y_1 &\leq 1-\frac{1-y}{1-x}\left(1-x_1\right) \\
&\leq 1-\frac{1-y}{1-x}\left(1-\frac{y_1}{y}\cdot x\right), \\
y_1 &\leq \frac{1-\frac{1-y}{1-x}}{1-\frac{1-y}{1-x}\cdot\frac{x}{y}}=y.
\end{aligned}$$

This, in turn, implies that

$$x_1 \leq x \text{ and } y_1 \leq y.$$

Following this deviation, persuader 1's expected utility becomes

$$\begin{aligned}
V_1 &= \frac{1}{2}\left[y_1\left(1-y\right)+x_1\left(1-x\right)+\left(1-y_1\right)y+\left(1-x_1\right)x+y_1y+x_1x\right] \\
&= \frac{1}{2}\left[\left(1-y\right)y_1+\left(1-x\right)x_1+x+y\right].
\end{aligned}$$

Since V_1 is increasing in both x_1 and y_1, V_1 is maximized when $x_1=x$ and $y_1=y$, i.e. there is no profitable deviation to some $\left(x_1,y_1\right)\neq\left(x,y\right)$ s.t. $A_a=\left\{\left\{1\right\},\left\{2\right\},\left\{1,2\right\}\right\}$.

(2) Suppose that persuader 1 deviates to some $\left(x_2,y_2\right)$ s.t. $A_2=\left\{\left\{2\right\},\left\{1,2\right\}\right\}$. This implies that the decision maker chooses a_H if and only if persuader 2's test result is positive.

Following this deviation, persuader 1's payoff becomes

$$\begin{aligned}
V_2 &= \frac{1}{2}\left(y+x\right) \\
&< \frac{1}{2}\left[y+x+\left(y-y^2\right)+\left(x-x^2\right)\right]=V.
\end{aligned}$$

Therefore, this deviation is not profitable for persuader 1.

(3) Suppose that persuader 1 deviates to some $\left(x_3,y_3\right)$ s.t. $A_3=\left\{\left\{1,2\right\}\right\}$. This implies that the decision maker chooses a_H if and only if both tests' results are positive. Persuader

1's payoff following this deviation is

$$\begin{aligned} V_3 &= \frac{1}{2}(y \cdot y_3 + x \cdot x_3) \\ &\leq \frac{1}{2}(y+x) = V_2 \\ &< V. \end{aligned}$$

Therefore, this deviation is not profitable for persuader 1.

(4) Finally, suppose that persuader 1 deviates to some (x_4, y_4) s.t. $A_4 = \{\{1\}, \{1,2\}\}$. This implies that x_4 and y_4 must satisfy

$$\frac{y_4}{x_4} \cdot \frac{1-y}{1-x} \geq \frac{y}{x} \cdot \frac{1-y}{1-x},$$

which, in turn, implies that

$$x_4 \leq \frac{x}{y} \text{ and } y_4 \leq 1.$$

Following this deviation, persuader 1's expected utility is

$$\begin{aligned} V_4 &= \frac{1}{2}(y_4 + x_4) \\ &\leq \frac{1}{2}\left(1 + \frac{x}{y}\right). \end{aligned}$$

A sufficient condition for $V_4 \leq V$ is

$$\frac{1}{2}\left(1 + \frac{x}{y}\right) \leq V.$$

This inequality can be written as

$$\left(2y - y^2 + 2x - x^2\right) - \left(1 + \frac{x}{y}\right) \geq 0.$$

Replacing y with $\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{p_d}{1-p_d}\left(x-x^2\right)}$, the left-hand side of the inequality becomes

$$\begin{aligned} LHS(x) \equiv & \left(\frac{p_d}{1-p_d}+1\right)\left(x-x^2\right) \\ & +x+\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{p_d}{1-p_d}\left(x-x^2\right)} \\ & -1-\frac{x}{\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{p_d}{1-p_d}\left(x-x^2\right)}}. \end{aligned}$$

Note that for all $p_d \in \left(\frac{1}{2},1\right)$, $LHS(0)=0$ and $LHS^{'}(0)=1$. This means that there exists some $\bar{x} \in \left(0, \frac{1}{2}-\frac{1}{2}\sqrt{2-\frac{1}{p_d}}\right)$ such that the deviation to (x_4, y_4) is not profitable whenever $x \in (0, \bar{x})$.

Summarizing cases (1) - (4), the symmetric test profile (x,y) does not have any profitable deviation whenever $x \in (0,\bar{x})$ and $y=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{p_d}{1-p_d}\left(x-x^2\right)}<1$. Because $y<1$, (x,y) constitutes an equilibrium with $U>0$ by Proposition 2. This suffices to prove Claim 1.

Claim 2: Let $n>2$. There is an $n-$persuader equilibrium in which $x_1=x_2=x$, $y_1=y_2=y<1$, and all of the other persuaders are uninformative with $x_i=y_i=1$ for all $i>2$. The decision maker's payoff is $U>0$ and her acceptance set is $A=\{\{1\},\{2\},\{1,2\}\}$ (she chooses a_H if and only if persuader 1 or 2's test result is positive).

Proof of Claim 2: Let $\bar{x}$ be defined as in the proof of Claim 1. Let the first two persuaders choose the test (x,y) such that $x \in (0,\bar{x})$ and $y=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{p_d}{1-p_d}\left(x-x^2\right)}<1$, and the rest of the persuaders choose uninformative tests with $x_i=y_i=1$ for all $i>2$. As shown in the proof of Claim 1, if the persuaders choose the proposed test profile, their payoff is

$$V=\frac{1}{2}\left(2y-y^2+2x-x^2\right).$$

Claim 1 implies that there is no profitable deviation for the first or the second persuader. It is sufficient to proof Claim 2 by showing that there always exists some (x,y) such that the third persuader cannot profitably deviate to any informative test $\left(x^{'},y^{'}\right)$ with $x^{'}<y^{'}$.

Recall that the decision maker is indifferent after one positive result and one negative result from the first two persuaders. Therefore, if the third persuader deviates to some informative test, it is impossible for the decision maker to choose a_H when the third persuader's result is negative, and only one of the first two persuaders' test results is positive. Moreover, if the third persuader's test result is positive and at least one result from the first two persuaders is also positive, then the decision maker must strictly prefer a_H. This implies that the decision maker's acceptance set never contains $\{1\}$ or $\{2\}$, and it always contains $\{1,3\}$, $\{2,3\}$, and $\{1,2,3\}$. Therefore, it is sufficient to check only the following three cases of deviation:

a. Let persuader 3 deviate to some $\left(x_a^{'}, y_a^{'}\right)$ such that the decision maker chooses a_H if and only if persuader 3's test result is positive (i.e., her acceptance set is $A_a^{'} = \{\{3\},\{1,3\},\{2,3\},\{1,2,3\}\}$). Among deviations of this type, the most profitable one leaves the decision maker indifferent when only persuader 3's test result is positive, which implies that persuader 3's deviation $\left(x_a^{'}, y_a^{'}\right)$ satisfies

$$\frac{(1-y)^2}{(1-x)^2} \cdot \frac{y_a^{'}}{x_a^{'}} = \frac{p_d}{1-p_d} = \frac{y}{x} \cdot \frac{1-y}{1-x},$$

$$\frac{y_a^{'}}{x_a^{'}} = \frac{y}{x} \cdot \frac{1-x}{1-y}.$$

Persuader 3's payoff is $V_a^{'} = \frac{1}{2}(1+x_a)$. $V_a^{'}$ is maximized when $y_a^{'} = 1$ and $x_a^{'} = \frac{x}{y} \cdot \frac{1-y}{1-x}$, in which case $V_a^{'} = \frac{1}{2}\left(1+\frac{x}{y} \cdot \frac{1-y}{1-x}\right)$. The proof of Claim 1 shows that $\frac{1}{2}\left(1+\frac{x}{y}\right) \leq V$. Since $\frac{1-y}{1-x} \in (0,1)$, $V_a^{'} < \frac{1}{2}\left(1+\frac{x}{y}\right) \leq V$. This deviation is strictly non-profitable.

b. Let persuader 3 deviate to some $\left(x_b^{'}, y_b^{'}\right)$ such that the decision maker's new acceptance set becomes $A_b^{'} = \{\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$. Persuader 3's new payoff is $V_b^{'} = \frac{1}{2}\left[2y(1-y)y_b^{'} + y^2 + 2x(1-x)x_b^{'} + x^2\right] < V$ for all $\left(x_b^{'}, y_b^{'}\right) \neq (1,1)$. This type of deviation is strictly non-profitable.

c. Let persuader 3 deviate to some $\left(x_c^{'}, y_c^{'}\right)$ such that the decision maker's new acceptance set becomes $A_b^{'} = \{\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$. This implies that $\left(x_c^{'}, y_c^{'}\right)$ must satisfy

$$\frac{(1-y)^2}{(1-x)^2} \cdot \frac{y_c^{'}}{x_c^{'}} \geq \frac{p_d}{1-p_d} \text{ and } \frac{\left(1-y_c^{'}\right)}{\left(1-x_c^{'}\right)} \cdot \frac{y^2}{x^2} \geq \frac{p_d}{1-p_d}.$$

These inequalities are equivalent to

$$\frac{y_c^{'}}{x_c^{'}} \geq \frac{y}{x} \cdot \frac{1-x}{1-y}, \tag{2}$$

and

$$\frac{1-y_c^{'}}{1-x_c^{'}} \geq \frac{x}{y} \cdot \frac{1-y}{1-x}. \tag{3}$$

Because persuader 3's payoff $V_c^{'} = \frac{1}{2}\left[y_c + x_c + (1-y_c)\, y^2 + (1-x_c)\, x^2\right]$ increases in $x_c^{'}$ and $y_c^{'}$, $V_c^{'}$ is maximized when both (2) and (3) hold with equality - i.e.,

$$\frac{y_c^{'}}{x_c^{'}} = \frac{y}{x} \cdot \frac{1-x}{1-y} \text{ and } \frac{1-y_c^{'}}{1-x_c^{'}} = \frac{x}{y} \cdot \frac{1-y}{1-x},$$

which yields

$$x_c^{'} = \frac{x-xy}{x+y-2xy} \text{ and } y_c^{'} = \frac{y-xy}{x+y-2xy}.$$

$$V_c^{'} = \frac{y + x^2 y - x^3 y + x\left(1-2y+y^2-y^3\right)}{2\,(x+y-2xy)}$$

$$V - V_c^{'} = \frac{x^3\,(1-3y) + (1-y)^2\, y + x^2\,(6y-2) + x\left(1-6y+6y^2-3y^3\right)}{-2y + x\,(4y-2)}$$

Replace y with $y(x) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}\,(x-x^2)}$ and express $V - V_c^{'}$ as a single-variable function of x. Define $h(x) \equiv V - V_c^{'}$ then,

$$h(x) = \frac{(1-x)x\left\{\left[\frac{p_d}{1-p_d}(3x-1) + 3(x-1)\right]\sqrt{1+4\frac{p_d}{1-p_d}(x-1)x} - \frac{p_d}{1-p_d}(3x-1) + x - 1\right\}}{(4x-2)\sqrt{1+4\frac{p_d}{1-p_d}(x-1)x} - 2}.$$

Note that $h(0) = 0$ and $h'(0) = 1$. In other words, there exists some $\hat{x} > 0$ such that $V - V_c^{'} > 0$ whenever $x \in (0, \hat{x})$, and no profitable deviation exists for the third persuader. Therefore, the proposed test profile constitutes an equilibrium whenever $x \in (0, \min\{\bar{x}, \hat{x}\})$. This completes the proof of Claim 2.

Claim 1 and 2 prove part (b) of Proposition 3.

It is worth noting that symmetric equilibrium with $U > 0$ also exists when $n > 2$. For example, when $n = 3$ and $p_d = \frac{81}{113}$, there exists an equilibrium in which all persuaders choose the test $(x_i, y_i) = (0.2, 0.9)$. The decision maker's acceptance fraction is $\frac{2}{3}$. $U > 0$ for any $d > 0$.

6.5 Proof of Remark 2

Given n, k, and $\bar{y}$, let $f(x) = \left(\frac{\bar{y}}{x}\right)^k \left(\frac{1-\bar{y}}{1-x}\right)^{n-k}$.

To prove that $f(x) = \frac{p_d}{1-p_d}$ has a solution on $(0,\bar{y})$, note that $\lim_{x\to 0} f(x) = \infty$ and $f(\bar{y}) = 1 < \frac{p_d}{1-p_d}$. Therefore, by the Intermediate Value Theorem, there must exist at least one $x^* \in (0,\bar{y})$ such that $f(x^*) = \frac{p_d}{1-p_d}$.

To prove that x^* is unique, note that

$$f^{'}(x) = \frac{(nx-k)\left(\frac{\bar{y}}{x}\right)^k \left(\frac{1-\bar{y}}{1-x}\right)^{n-k}}{(1-x)x} \begin{cases} >0 & \text{when } x > \frac{k}{n} \\ =0 & \text{when } x = \frac{k}{n} \\ <0 & \text{when } x < \frac{k}{n} \end{cases}$$

If $\bar{y} \le \frac{k}{n}$, then $f(x)$ is strictly decreasing for all $x \in (0,\bar{y})$ and $f(x) = \frac{p_d}{1-p_d}$ must have a unique solution on this domain.

Suppose that $\bar{y} > \frac{k}{n}$. Since f is increasing on $\left[\frac{k}{n},\bar{y}\right]$ and $f(\bar{y}) < \frac{p_d}{1-p_d}$, the solution of $f(x) = \frac{p_d}{1-p_d}$ cannot lie in the interval $\left[\frac{k}{n},\bar{y}\right]$. Hence, $x^* \in \left(0,\frac{k}{n}\right)$. Since f is strictly decreasing on $\left(0,\frac{k}{n}\right)$, x^* is unique.

To show that x^* strictly increases in k, first note that, given $\bar{y}$ and p_d, Equation (1) implicitly defines x^* as a function of k. Differentiating both sides of Equation (1) yields

$$f^{'}(x^*) \cdot \frac{dx^*}{dk} + f^{'}(k) = 0.$$

$f^{'}(x^*) < 0$ as shown above. Fixing x and $\bar{y}$, f strictly increases in k, so $f^{'}(k) > 0$. This implies that $\frac{dx^*}{dk} = -f^{'}(k)/f^{'}(x^*) > 0$.

6.6 Proof of Lemma 1

Proof of (a):

In a symmetric binding test profile with acceptance fraction $\frac{k}{n}$, persuaders choose the test $(x^*,\bar{y})$ such that

$$\left(\frac{\bar{y}}{x^*}\right)^k \left(\frac{1-\bar{y}}{1-x^*}\right)^{n-k} = \frac{p_d}{1-p_d}.$$

Suppose that persuader i deviates to an arbitrary test $\left(x^{'},y^{'}\right)$ such that $x^{'} \le y^{'} \le \bar{y}$. It must be the case that

$$\frac{1-y^{'}}{1-x^{'}} < \frac{\bar{y}}{x^{*}}$$

because $x^{'} \leq y^{'}$ by construction and $x^{*} < \bar{y}$ (Remark 2), which implies that $\frac{1-y^{'}}{1-x^{'}} \leq 1 < \frac{\bar{y}}{x^{*}}$.

Therefore, when persuader i's test result is negative and only $k-1$ out of the other $n-1$ persuaders' test results are positive, the posterior likelihood ratio is

$$\begin{aligned}\left(\frac{1-y^{'}}{1-x^{'}}\right)\left(\frac{\bar{y}}{x^{*}}\right)^{k-1}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-k} &< \left(\frac{\bar{y}}{x^{*}}\right)^{k}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-k} \\ &= \frac{p_d}{1-p_d}.\end{aligned}$$

The decision maker is unwilling to choose a_H.

This means that, regardless of the deviation, if i's test result is negative, the decision maker is unwilling to decrease the number of positive results required for a_H. In other words, c_1 must be weakly positive.

Proof of (b): I prove $c_2 < 0$ in two steps. First, I show that c_2 cannot be strictly positive. Then, I show that $c_1 \geq 0$ implies that $c_2 \neq 0$.

Claim 1: If $c_1 \geq 0$ and $c_2 > 0$, then the deviation $\left(x^{'}, y^{'}\right)$ cannot be profitable. Therefore, $c_1 \geq 0$ implies that $c_2 \leq 0$.

Proof. Suppose that, when i's test result is negative, the decision maker requires $k+c_1$ or more positive results from the other non-deviating persuaders in order to choose a_H. Meanwhile, when i's test result is positive, the decision maker still requires $k-1+c_2$ or more positive results to choose a_H. (If the decision maker never chooses a_H when i's test result is negative, c_1 can be any integer strictly greater than $n-1-k$; if the decision maker never chooses a_H when i's test result is positive, c_2 can be any integer strictly greater than $n-k$.)

Let $p =$ the unconditional probability of the event "there are at least k positive test results from $n-1$ persuaders with tests $(x^{*}, \bar{y})$."

Let $V^{'}$ denote the unconditional probability of a_H when i deviates to $\left(x^{'}, y^{'}\right)$ and the other persuaders choose tests $(x^{*}, \bar{y})$.

If $c_1 \geq 0$ and $c_2 > 0$ then $k + c_1 \geq k$ and $k - 1 + c_2 \geq k$ (recall that c_1, c_2 are integers). In other words, regardless of i's test result, a necessary condition for the decision maker to choose a_H is that there must be at least k positive results from the other $n-1$ non-deviating persuaders with tests $(x^*, \bar{y})$. This implies that $V' \leq p$.

Let V^* denote the unconditional probability of a_H when all persuaders, including i, choose $(x^*, \bar{y})$. Because $(x^*, \bar{y})$ induces the acceptance fraction $\frac{k}{n}$ by assumption, V^* is simply the unconditional probability of the event "there are at least k positive test results from n persuaders with tests $(x^*, \bar{y})$." This event is a necessary but not sufficient condition for the event "there are at least k positive test results from the first $n-1$ persuaders with tests $(x^*, \bar{y})$." Therefore, $V^* > p$. Because $V' \leq p$, this implies that $V' < V^*$, i.e., i's deviation to $\left(x', y'\right)$ is strictly unprofitable when $c_1 \geq 0$ and $c_2 > 0$.

Therefore, for i's deviation to be profitable, $c_1 \geq 0$ must imply that $c_2 \leq 0$. □

Claim 2: If $c_1 \geq 0$ and $c_2 = 0$, then the deviation $\left(x', y'\right)$ cannot be profitable. Therefore, $c_1 \geq 0$ implies that $c_2 \neq 0$.

Proof. Suppose that $c_1 \geq 0$ and $c_2 = 0$. The fact that $c_2 = 0$ implies that

$$\frac{y'}{x'} \cdot \left(\frac{\bar{y}}{x^*}\right)^{k-1} \left(\frac{1-\bar{y}}{1-x^*}\right)^{n-k} \geq \frac{p_d}{1-p_d} = \left(\frac{\bar{y}}{x^*}\right)^{k} \left(\frac{1-\bar{y}}{1-x^*}\right)^{n-k}$$
$$\frac{y'}{x'} \geq \frac{\bar{y}}{x^*}$$

Note that $y' \leq \bar{y}$ by construction, so the inequality above implies that $x' \leq \frac{y'}{\bar{y}} \cdot x^* \leq x^*$. In other words, compared to the original test $(x^*, \bar{y})$, i's test $\left(x', y'\right)$ has a lower probability of yielding a positive result in either state. Moreover, because $c_1 \geq 0$ and $c_2 = 0$, the decision maker requests seeing weakly more counts of positive results from the n persuaders in order to choose a_H. This combined with the low $\left(x', y'\right)$ implies that, after i's deviation, it is less likely for there to be enough counts of positive results to meet the decision maker's increased acceptance fraction. Therefore, the deviation to $\left(x', y'\right)$ weakly decreases the unconditional probability of a_H if this deviation induces $c_1 \geq 0$ and $c_2 = 0$. If $c_1 \geq 0$ and the deviation is indeed profitable, it must be the case that $c_2 \neq 0$. □

Because c_1 is always non-negative, Claims 1 and 2 imply that if i's deviation is profitable, c_2 must be strictly negative, i.e., when i's test result is positive, the decision maker is willing to choose a_H upon seeing strictly fewer counts of positive results from the other non-deviating persuaders.

6.7 Most profitable deviations from the symmetric and binding test profile with $\alpha = 1$

Lemma 4. *Given $n \geq 2$, $p_d \in \left(\frac{1}{2}, 1\right)$, and $\bar{y} \in (0,1)$, suppose that all n persuaders start with the symmetric and binding test profile $(x^*, \bar{y})$ with acceptance fraction $\alpha = 1$. Consider unilateral deviations from persuader i to some arbitrary test design $\left(x', y'\right) \neq (x^*, \bar{y})$ such that when i's test result is positive, the decision maker chooses a_H if and only if there are at least $n-1-c$ positive results from the other $n-1$ non-deviating persuaders for some $1 \leq c \leq n-1$.*[8]

For each c, the most profitable deviation of this kind is

$$
\begin{aligned}
x'(c) &= \bar{y}\left[\left(\frac{\bar{y}}{x^*}\right)^{c+1}\left(\frac{1-x^*}{1-\bar{y}}\right)^{c}\right]^{-1}, \\
y'(c) &= \bar{y}.
\end{aligned}
$$

Moreover, the symmetric and binding test profile $(x^*, \bar{y})$ with acceptance fraction $\alpha = 1$ constitutes an equilibrium if and only if the deviation $\left(x'(c), y'(c)\right)$ is not profitable for any c such that $1 \leq c \leq n-1$.

Proof. First of all, regardless of *i*'s deviation, if *i*'s result is negative, the decision maker never chooses a_H because positive results from the original tests are not strong enough. Lemma 1 indicates that if a profitable deviation exists, a positive result from the deviant must be able to offset strictly more negative results from the non-deviating persuaders. Hence, this Lemma focuses on the case of $c \geq 1$.

Suppose that when *i*'s test result is positive, the decision maker's best response is to choose a_H if and only if there are at least $n-1-c$ positive results from the other $n-1$ non-deviating persuaders, where c is an exogenous integer between 1 and $n-1$. Here, among all deviations that trigger this type of best response, I calculate the most profitable one for persuader *i*, which I call $\left(x'(c), y'(c)\right)$.

[8] The integer c here replaces the integer $-c_2$ in Lemma 1.

Given c, i's deviation $\left(x^{'},y^{'}\right)$ must satisfy

$$\begin{aligned}
\frac{y^{'}}{x^{'}}\cdot\left(\frac{\bar{y}}{x^{*}}\right)^{n-1-c}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{c} &\geq \frac{p_d}{1-p_d}=\left(\frac{\bar{y}}{x^{*}}\right)^{n}\\
\frac{y^{'}}{x^{'}} &\geq \left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\\
x^{'} &\leq y^{'}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\right]^{-1}\\
&\leq \bar{y}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\right]^{-1} \text{ (equality holds when } y^{'}=\bar{y}\text{)}
\end{aligned}$$

Meanwhile, i's expected utility after deviation is

$$\begin{aligned}
V_{dev}\left(x^{'},y^{'},c\right) &= \Pr(\text{positive result from } i \text{ and at least } n-1-c \text{ other persuaders})\\
&= \frac{1}{2}\sum_{j=n-1-c}^{n-1}\binom{n-1}{j}\left[y^{'}\bar{y}^{j}\left(1-\bar{y}\right)^{n-1-j}+x^{'}\left(x^{*}\right)^{j}\left(1-x^{*}\right)^{n-1-j}\right],
\end{aligned}$$

which strictly increases in both $x^{'}$ and $y^{'}$. Because $y^{'}\leq\bar{y}$ and $x^{'}\leq\bar{y}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\right]^{-1}$, $V_{dev}\left(x^{'},y^{'},c\right)$ is maximized exactly when $y^{'}=\bar{y}$ and $x^{'}=\bar{y}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\right]^{-1}\in(0,\bar{y})$, in which case the decision maker is indifferent between a_H and a_L when she sees positive test results from i and $n-1-c$ other non-deviating persuaders. This proves $\left(x^{'}(c),y^{'}(c)\right)=\left(\bar{y}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{c+1}\left(\frac{1-x^{*}}{1-\bar{y}}\right)^{c}\right]^{-1},\bar{y}\right)$.

Finally, Lemma 1 implies that any deviation that is not described in Lemma 4 is not profitable. Therefore, if the deviation $\left(x^{'}(c),y^{'}(c)\right)$ is not profitable for any c such that $1\leq c\leq n-1$, the symmetric and binding test profile $(x^{*},\bar{y})$ with acceptance fraction 1 constitutes an equilibrium. This completes the proof of the last part of Lemma 4. □

6.8 Most profitable deviations from the symmetric and binding test profile with $\alpha<1$

Lemma 5. *Given $1<k<n$, $p_d\in\left(\frac{1}{2},1\right)$, and $\bar{y}\in(0,1)$, let c_1 and c_2 be integers. Suppose that all n persuaders start with the symmetric and binding test profile $(x^{*},\bar{y})$ with acceptance fraction $\frac{k}{n}$. Consider a unilateral deviation from persuader i to some arbitrary test*

design $\left(x^{'},y^{'}\right)\neq(x^{*},\bar{y})$ *such that the minimum number of positive results required from the non-deviating persuaders in order to induce* a_H *increases by*

(a) $c_1\in[0,n-1-k]$ *when i's test result is negative, and*

(b) $c_2\in[-k+1,-1]$ *when i's test result is positive.*

Then, among all possible deviations $\left(x^{'},y^{'}\right)$ *that satisfy conditions (a) and (b), the one that maximizes persuader i's expected utility is*

$$\begin{aligned} y^{'}(c_1,c_2) &= \min\left\{\bar{y},B(c_2)\cdot\frac{1-A(c_1)}{B(c_2)-A(c_1)}\right\}, \\ x^{'}(c_1,c_2) &= \frac{y^{'}(c_1,c_2)}{B(c_2)} \end{aligned}$$

where $A(c_1)=\left(\frac{\bar{y}}{x^{*}}\right)^{-c_1}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{1+c_1}$ *and* $B(c_2)=\left(\frac{\bar{y}}{x^{*}}\right)^{1-c_2}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{c_2}$.

Moreover, the symmetric and binding test profile $(x^{*},\bar{y})$ *with acceptance fraction* $\frac{k}{n}$ *constitutes an equilibrium if and only if the deviation* $\left(x^{'}(c_1,c_2),y^{'}(c_1,c_2)\right)$ *is not profitable for any* c_1, c_2 *such that* $0\leq c_1\leq n-1-k$ *and* $-k+1\leq c_2\leq -1$.

Proof. Recall that the symmetric and binding test profile $(x^{*},\bar{y})$ with acceptance fraction $\frac{k}{n}$ satisfies the following condition:

$$\left(\frac{\bar{y}}{x^{*}}\right)^{k}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-k}=\frac{p_d}{1-p_d}.$$

If a deviation $\left(x^{'},y^{'}\right)$ satisfies conditions (a) and (b) in Lemma 5, then it must satisfy

$$\begin{aligned} \left(\frac{1-y^{'}}{1-x^{'}}\right)\left(\frac{\bar{y}}{x^{*}}\right)^{k+c_1}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-1-k-c_1} &\geq \frac{p_d}{1-p_d}, \\ &\text{and} \\ \left(\frac{y^{'}}{x^{'}}\right)\left(\frac{\bar{y}}{x^{*}}\right)^{k-1+c_2}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-k-c_2} &\geq \frac{p_d}{1-p_d}. \end{aligned}$$

Replacing $\frac{p_d}{1-p_d}$ with $\left(\frac{\bar{y}}{x^{*}}\right)^{k}\left(\frac{1-\bar{y}}{1-x^{*}}\right)^{n-k}$, this is equivalent to

$$\begin{aligned}\frac{1-y'}{1-x'} &\geq \left(\frac{\bar{y}}{x^*}\right)^{-c_1}\left(\frac{1-\bar{y}}{1-x^*}\right)^{1+c_1}, \\ &\text{and} \\ \frac{y'}{x'} &\geq \left(\frac{\bar{y}}{x^*}\right)^{1-c_2}\left(\frac{1-\bar{y}}{1-x^*}\right)^{c_2}.\end{aligned}$$

Let $A(c_1) = \left(\frac{\bar{y}}{x^*}\right)^{-c_1}\left(\frac{1-\bar{y}}{1-x^*}\right)^{1+c_1}$ and $B(c_2) = \left(\frac{\bar{y}}{x^*}\right)^{1-c_2}\left(\frac{1-\bar{y}}{1-x^*}\right)^{c_2}$. The pair of inequalities above is equivalent to

$$B(c_2)x' \leq y' \leq 1 - A(c_1)\left(1-x'\right).$$

Note that the deviating persuader's expected utility strictly increases in both x' and y'. Therefore, given c_1 and c_2, the deviation that gives the highest expected utility maximizes x' and y' subject to $B(c_2)x' \leq y' \leq 1 - A(c_1)\left(1-x'\right)$. This inequality constraint is well-defined only when $B(c_2)x' \leq 1 - A(c_1)\left(1-x'\right)$, which implies that $x' \leq \frac{1-A(c_1)}{B(c_2)-A(c_1)}$. Moreover, both $B(c_2)x'$ and $1 - A(c_1)\left(1-x'\right)$ strictly increase in x'. These two observations imply that:

If $B(c_2) \cdot \frac{1-A(c_1)}{B(c_2)-A(c_1)} \leq \bar{y}$, then x' and y' are maximized when $x' = \frac{1-A(c_1)}{B(c_2)-A(c_1)}$ and $y' = B(c_2)x'$.

If $B(c_2) \cdot \frac{1-A(c_1)}{B(c_2)-A(c_1)} > \bar{y}$, then y' is maximized at $y' = \bar{y}$. Subject to the constraint $B(c_2)x' \leq y'$, x' is maximized at $x' = \frac{\bar{y}}{B(c_2)}$.

Therefore, given c_1 and c_2, the optimal deviation is

$$\begin{aligned}y'(c_1,c_2) &= \min\left\{\bar{y}, B(c_2) \cdot \frac{1-A(c_1)}{B(c_2)-A(c_1)}\right\}, \\ x'(c_1,c_2) &= \frac{y'(c_1,c_2)}{B(c_2)}.\end{aligned}$$

$\left(x'(c_1,c_2), y'(c_1,c_2)\right)$ is a well-defined deviation if $x'(c_1,c_2) < y'(c_1,c_2)$. This is always true because $c_2 < 0$ and $B(c_2) > 1$.

Finally, Lemma 1 implies that any deviation that does not satisfy condition (a) or (b) is not profitable. Therefore, if the deviation $\left(x'(c_1,c_2), y'(c_1,c_2)\right)$ is not profitable for any c_1, c_2 such that $0 \leq c_1 \leq n-1-k$ and $-k+1 \leq c_2 \leq -1$, then the symmetric and binding test profile $(x^*, \bar{y})$ constitutes an equilibrium with acceptance fraction $\frac{k}{n}$. This completes the proof. $\square$

6.9 Proof of Proposition 4

The "if" part: if an equilibrium induces $\alpha = 1$, it means that the decision maker's acceptance set is $A = \{N_I\}$. By Proposition 1, this implies that the decision maker's equilibrium payoff is $U = 0$.

The "only if" part: prove by contradiction. Recall that in a symmetric binding equilibrium with $\alpha = 1$, every persuader chooses $(x^*, \bar{y}) \equiv \left(\bar{y}\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}, \bar{y}\right)$. Suppose that there exists a non-beneficial symmetric equilibrium in which all persuaders choose some $(x,y) \neq (x^*, \bar{y})$. By Proposition 1, if this equilibrium is non-beneficial, it must induce the acceptance set $A = \{N_I\}$. This implies two things: (1) the persuaders' common expected utility is $V(x,y) = \frac{1}{2}(y^n + x^n)$, which strictly increases in both x and y. (2) (x,y) must satisfy $\left(\frac{y}{x}\right)^n \geq \frac{p_d}{1-p_d} = \left(\frac{\bar{y}}{x^*}\right)^n$ or $\frac{y}{x} \geq \frac{\bar{y}}{x^*}$. Since $y \leq \bar{y}$ (by construction) and $(x,y) \neq (x^*, \bar{y})$, (2) implies that $x < x^*$. Now, suppose that persuader 1 deviates to $(x^*, \bar{y})$. After this deviation, the decision maker is still willing to choose a_H when all test results are positive because $\frac{\bar{y}}{x^*}\left(\frac{y}{x}\right)^{n-1} \geq \left(\frac{\bar{y}}{x^*}\right)^n = \frac{p_d}{1-p_d}$. Therefore, persuader 1's expected utility becomes $\frac{1}{2}\left(\bar{y}y^{n-1} + x^* x^{n-1}\right) > V(x,y)$. Hence, this is a profitable deviation and the proposed strategy profile (x,y) cannot be an equilibrium.

6.10 Proof of Proposition 5

Let $k_1 < k_2$. Let $(x_1^*, \bar{y})$ and $(x_2^*, \bar{y})$ be the corresponding tests in the symmetric binding equilibria with acceptance fractions $\frac{k_1}{n}$ and $\frac{k_2}{n}$, respectively. Remark 2 implies that $x_1^* < x_2^*$. Now, let's compare the equilibrium outcomes from the perspective of the decision maker. In the first equilibrium, she picks the best action based on information from n tests with conditional probabilities $(x_1^*, \bar{y})$. In the second equilibrium, she picks the best action based on information from n tests with conditional probabilities $(x_2^*, \bar{y})$. Since $x_1^* < x_2^*$, the test $(x_1^*, \bar{y})$ has a strictly lower false-positive probability and the same false-negative probability compared to the test $(x_2^*, \bar{y})$. In other words, each test in the equilibrium associated with k_1 is strictly more informative than each test in the equilibrium associated with k_2, and the decision maker's posterior belief distribution in the equilibrium with k_1 is a mean-preserving spread of the one in the equilibrium with k_2. Because the decision maker chooses both

a_L and a_H with positive probabilities in both equilibria, this spread of her posterior belief implies that her expected utility is strictly higher in the binding equilibrium associated with k_1.

6.11 Proof of Theorem 1

By Proposition 4, it suffices to prove Theorem 1 by showing that the symmetric binding equilibrium with $\alpha = 1$ does not exist when $\bar{y}$ is sufficiently low or when n is sufficiently large. Lemma 4 shows that the candidates for the most profitable unilateral deviation are tests $\left\{ \left(x^{'}(c), y^{'}(c) \right) \mid c = 1, ..., n-1 \right\}$ such that

$$
\begin{aligned}
x^{'}(c) &= \bar{y} \left[\left(\frac{\bar{y}}{x^*} \right)^{c+1} \left(\frac{1-x^*}{1-\bar{y}} \right)^{c} \right]^{-1} \\
y^{'}(c) &= \bar{y}
\end{aligned}
$$

It suffices to prove that the symmetric binding equilibrium with $\alpha = 1$ does not exist if one can show that at least one of these deviations $\left(x^{'}(c), y^{'}(c) \right)$ is strictly profitable. In the remainder of the proof, I focus on the case of $c = 1$ and show that the deviation to $\left(x^{'}(1), y^{'}(1) \right)$ is strictly profitable when $\bar{y}$ is sufficiently low or when n is sufficiently large. (Numerically, for $n \leq 20$ and any $(p_d, \bar{y})$, $\left(x^{'}(1), y^{'}(1) \right)$ is profitable for the widest range of $\bar{y}$ among all deviations in $\left\{ \left(x^{'}(c), y^{'}(c) \right) \mid c = 1, ..., n-1 \right\}$.)

Suppose that persuaders $2, 3, ..., n$ all choose the symmetric binding test with $\alpha = 1$:

$$
(x^*, \bar{y}) \text{ s.t. } x^* = \bar{y} \left(\frac{1-p_d}{p_d} \right)^{\frac{1}{n}}.
$$

If persuader 1 also chooses the same test $(x^*, \bar{y})$, then his payoff is

$$
\begin{aligned}
V(p_d, \bar{y}, n) &= \frac{1}{2} \left[\bar{y}^n + (x^*)^n \right] \\
&= \frac{1}{2} \left[\bar{y}^n + \bar{y}^n \left(\frac{1-p_d}{p_d} \right) \right] \\
&= \frac{\bar{y}^n}{2p_d}.
\end{aligned}
$$

If persuader 1 chooses a different test $\left(x^{'}(1), y^{'}(1)\right)$ such that

$$\begin{aligned} x^{'}(1) &= \bar{y}\left[\left(\frac{\bar{y}}{x^{*}}\right)^{2}\left(\frac{1-x^{*}}{1-\bar{y}}\right)\right]^{-1}, \\ y^{'}(1) &= \bar{y}, \end{aligned}$$

then the decision maker chooses a_H if and only if persuader 1 and at least $n-2$ out of the other $n-1$ non-deviating persuaders' test results are positive. Persuader 1's ex-ante expected payoff is

$$\begin{aligned} & V_{dev}(p_d, \bar{y}, n) \\ = \; & \frac{1}{2}\left[(n-1)\bar{y}^{n-1}(1-\bar{y}) + \bar{y}^{n} + (n-1)x^{'}(1)(x^{*})^{n-2}(1-x^{*}) + x^{'}(1)(x^{*})^{n-1}\right]. \end{aligned}$$

Define

$$\begin{aligned} R(p_d, \bar{y}, n) &:= \frac{V_{dev}(p_d, \bar{y}, n)}{V(p_d, \bar{y}, n)} \\ &= \frac{p_d + (n-2)(1-\bar{y}) - \frac{(1-p_d)(1-\bar{y})}{\left(\frac{1}{p_d}-1\right)^{\frac{1}{n}}\bar{y}-1}}{\bar{y}}. \end{aligned}$$

If $R(p_d, \bar{y}, n) > 1$, then the symmetric binding equilibrium with $\alpha = 1$ does not exist. With the following four claims, I show that this happens when $\bar{y}$ is sufficiently low or when n is sufficiently large.

Claim 1: $R(p_d, \bar{y}, n)$ strictly increases in n.

Proof. As n increases, $(n-2)(1-\bar{y})$ strictly increases. Moreover, since $p_d \in \left(\frac{1}{2}, 1\right)$, $\frac{1}{p_d} - 1 \in (0,1)$, so $\left(\frac{1}{p_d}-1\right)^{\frac{1}{n}}$ strictly increases in n. This implies that $-\frac{(1-p_d)(1-\bar{y})}{\left(\frac{1}{p_d}-1\right)^{\frac{1}{n}}\bar{y}-1}$ strictly increases in n and, therefore, $R(p_d, \bar{y}, n)$ strictly increases in n. □

Claim 2: $\forall \bar{y} \in (0,1)$, $p_d \in \left(\frac{1}{2}, 1\right)$, there exists some $N \geq 2$ such that $R(p_d, \bar{y}, n) > 1$ for all $n \geq N$.

Proof. For all $\bar{y} \in (0,1)$, $p_d \in \left(\frac{1}{2}, 1\right)$, when $n \to \infty$, $\lim_{n\to\infty} R(p_d, \bar{y}, n) = \infty$. Since $R(p_d, \bar{y}, n)$ strictly increases in n, there must exist some finite $N \geq 2$ such that $R(p_d, \bar{y}, n) > 1$ for all $n \geq N$. □

Claim 3: $R(p_d,\bar{y},n)$ strictly decreases in $\bar{y}$.

Proof. Note that

$$\frac{\partial R}{\partial \bar{y}} = \frac{1-n-\left(\frac{1-p_d}{p_d}\right)^{\frac{2}{n}}(n-2+p_d)\bar{y}^2+\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\bar{y}\left[2n-2-(1-p_d)\bar{y}\right]}{\bar{y}^2\left[\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\bar{y}-1\right]^2}$$

I break down this expression to determine its sign. Let

$$\begin{aligned} & R_{top}(p_d,\bar{y},n) \\ = \; & 1-n-\left(\frac{1-p_d}{p_d}\right)^{\frac{2}{n}}(n-2+p_d)\bar{y}^2+\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\bar{y}\left[2n-2-(1-p_d)\bar{y}\right], \end{aligned}$$

then, since

$$\frac{\partial R}{\partial \bar{y}} = \frac{R_{top}(p_d,\bar{y},n)}{\bar{y}^2\left[\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\bar{y}-1\right]^2}$$

and $\bar{y}^2\left[\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\bar{y}-1\right]^2$ is strictly positive,

$$\frac{\partial R}{\partial \bar{y}} < 0 \text{ if and only if } R_{top}(p_d,\bar{y},n) < 0.$$

In the following steps, to show that $R_{top}(p_d,\bar{y},n) < 0$, I first show that it strictly increases in $\bar{y}$ for all p_d and n. Then, I show that $R_{top}(p_d,1,n)$ strictly decreases in p_d for all n. Therefore, since $p_d > \frac{1}{2}$, $R_{top}(p_d,\bar{y},n) < R_{top}\left(\frac{1}{2},1,n\right) = 0$.

To show that R_{top} strictly increases in $\bar{y}$, calculate

$$\begin{aligned} & \frac{\partial R_{top}}{\partial \bar{y}} \\ = \; & 2\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\left[n-1-(1-p_d)\bar{y}-\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}(n-2+p_d)\bar{y}\right]. \end{aligned}$$

Let

$$Q(p_d,\bar{y},n) = n-1-(1-p_d)\bar{y}-\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}(n-2+p_d)\bar{y}.$$

Then, since $\frac{\partial R_{top}}{\partial \bar{y}} = 2\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}} Q\left(p_d, \bar{y}, n\right)$ and $\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}} > 0$,

$$\frac{\partial R_{top}}{\partial \bar{y}} > 0 \text{ if and only if } Q\left(p_d, \bar{y}, n\right) > 0.$$

Note that Q strictly decreases in $\bar{y}$, so $Q\left(p_d, \bar{y}, n\right) > Q\left(p_d, 1, n\right)$ for all $n \geq 2$, $\bar{y} < 1$, and $p_d \in \left(\frac{1}{2}, 1\right)$. Also note that

$$\frac{\partial Q\left(p_d, 1, n\right)}{\partial p_d} = 1 - \left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}} + \frac{\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}-1}\left(n-2+p_d\right)}{n p_d^2} > 0.$$

Therefore,

$$Q\left(p_d, \bar{y}, n\right) > Q\left(p_d, 1, n\right) > Q\left(\frac{1}{2}, 1, n\right) = 0.$$

This implies that $\frac{\partial R_{top}}{\partial \bar{y}} > 0$. Therefore, $R_{top}\left(p_d, \bar{y}, n\right) < R_{top}\left(p_d, 1, n\right)$ for all $n \geq 2$, $\bar{y} < 1$, and $p_d \in \left(\frac{1}{2}, 1\right)$.

To show that $R_{top}\left(p_d, 1, n\right)$ strictly decreases in p_d, calculate

$$\begin{aligned} & \frac{\partial R_{top}\left(p_d, 1, n\right)}{\partial p_d} \\ = \; & -\frac{\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}}{n\left(1-p_d\right)p_d}\left\{\left[n\left(2-p_d+p_d^2\right)-4+2p_d\right]\left[1-\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}\right]+\left(1-p_d\right)\right\}. \end{aligned}$$

Note that

$$\begin{aligned} -\frac{\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}}{n\left(1-p_d\right)p_d} &< 0, \\ n\left(2-p_d+p_d^2\right)-4+2p_d &\geq 2\left(2-p_d+p_d^2\right)-4+2p_d \\ &= 2p_d^2 \\ &> 0, \\ 1-\left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}} &\geq 0, \\ 1+p_d &> 0. \end{aligned}$$

Therefore, $\frac{\partial R_{top}(p_d,1,n)}{\partial p_d} < 0$ and $R_{top}\left(p_d, \bar{y}, n\right) < R_{top}\left(\frac{1}{2}, 1, n\right)$ for all $n \geq 2$, $\bar{y} < 1$, and

$p_d \in \left(\frac{1}{2},1\right)$. Moreover, since

$$R_{top}\left(\frac{1}{2},1,n\right)=0,$$

$R_{top}\left(p_d,\bar{y},n\right)<0$ for all $\bar{y}<1$, $p_d \in \left(\frac{1}{2},1\right)$ and $n \geq 2$. This completes the proof for

$$\frac{\partial R}{\partial \bar{y}}<0.$$

□

Claim 4: $\forall n \geq 2$, $p_d \in \left(\frac{1}{2},1\right)$, there exists some $Y \in (p_d,1)$ such that $R\left(p_d,\bar{y},n\right)>1$ if and only if $\bar{y}<Y$.

Proof. Recall that

$$R\left(p_d,\bar{y},n\right)=\frac{p_d+(n-2)\left(1-\bar{y}\right)-\frac{(1-p_d)(1-\bar{y})}{\left(\frac{1}{p_d}-1\right)^{\frac{1}{n}}\bar{y}-1}}{\bar{y}}.$$

$\forall \bar{y} \in (0,1)$, $p_d \in \left(\frac{1}{2},1\right)$, $\lim_{\bar{y}\to 1} R\left(p_d,\bar{y},n\right)=p_d<1$. When $\bar{y}=p_d$,

$$R\left(p_d,p_d,n\right) \geq R\left(p_d,p_d,2\right)=\frac{p_d+\frac{(1-p_d)^2}{1-\sqrt{p_d(1-p_d)}}}{p_d}>1.$$

Since R is strictly decreasing in $\bar{y}$, this implies that there exists some $Y \in (p_d,1)$ such that $R\left(p_d,\bar{y},n\right)>1$ if and only if $\bar{y}<Y$. □

Finally, Claim 2 proves that $\forall p_d \in \left(\frac{1}{2},1\right)$ and $\bar{y} \in (0,1)$, there exists some $N \geq 2$ s.t. there does not exist any non-beneficial symmetric equilibrium if $n \geq N$. Claim 1 and 3 imply that this value N must increase in $\bar{y}$. The proof of Claim 4 also shows that when $\bar{y} \leq p_d$, $N\left(\bar{y}\right)=2$.

Claim 4 proves that $\forall p_d \in \left(\frac{1}{2},1\right)$ and $n \geq 2$, there exists some $Y>p_d$ s.t. there does not exist any non-beneficial symmetric equilibrium if $\bar{y}<Y$. Claim 1 and 3 imply that this value Y must increase in n. The proof of Claim 2 also shows that when $n \to \infty$, $Y \to 1$. This completes the proof of Theorem 1.

It is worth noting that the benefit from deviating to a more informative test is not monotonic in p_d. Therefore, neither N nor Y in Theorem 1 is monotonic in p_d.

6.12 Proof of Theorem 2

Given a symmetric binding test profile with acceptance fraction $\frac{1}{n}$, if persuader 1's test result is positive, the decision maker chooses a_H even when none of the other $n-1$ persuaders' tests yields a positive result. Suppose that persuader 1 deviates to a different test. Upon seeing a positive result from persuader 1's new test, suppose that the decision maker requires to see at least c_2 more positive results in order to choose a_H. By Lemma 1, if persuader 1's deviation is profitable, c_2 must be strictly lower than 0, which is technically impossible. Therefore, the symmetric binding test profile with acceptance fraction $\frac{1}{n}$ is always an equilibrium because it does not have any profitable deviation.

By Propositions 4 and 5, because this symmetric binding equilibrium induces the lowest acceptance fraction, it is associated with the highest payoff for the decision maker and this payoff is strictly higher than 0.

6.13 Proof of Proposition 6

By Proposition 5, for each n, the symmetric binding equilibrium that maximizes the decision maker's payoff is the one with acceptance fraction $\frac{1}{n}$. In this equilibrium, the persuaders choose test $(x_n^*, \bar{y})$ such that

$$f\left(x_n^*\right) \equiv \left(\frac{\bar{y}}{x_n^*}\right)\left(\frac{1-\bar{y}}{1-x_n^*}\right)^{n-1} = \frac{p_d}{1-p_d}.$$

Differentiating both sides of this equation with respect to n yields

$$f^{'}\left(x_n^*\right)\cdot\frac{dx_n^*}{dn} + f^{'}(n) = 0.$$

The proof in Remark 2 has shown that $f^{'}\left(x_n^*\right) < 0$. Fixing x_n^* and $\bar{y}$, f strictly decreases in n, so $f^{'}(n) < 0$. This implies that $\frac{dx_n^*}{dn} = -f^{'}(n)/f^{'}\left(x_n^*\right) < 0$. Because x_n^* is the false-positive probability, this means that the informativeness of the test $(x_n^*, \bar{y})$ strictly increases with n.

Given arbitrary positive integers $n_1 < n_2$, $U^*(n_1)$ is the decision maker's ex-ante expected utility when she best responds to n_1 symmetric independent tests $\left(x_{n_1}^*, \bar{y}\right)$, and $U^*(n_2)$ is her ex-ante expected utility when she best responds to n_2 symmetric independent tests $\left(x_{n_2}^*, \bar{y}\right)$. In the latter case, the decision maker learns from a greater number of tests (because $n_2 > n_1$) and each of these tests is strictly more informative (because $x_{n_2}^* < x_{n_1}^*$).

Therefore, the decision maker's posterior belief distribution in the equilibrium with n_2 is a mean-preserving spread of the one in the equilibrium with n_1. Because the decision maker chooses both a_L and a_H with positive probabilities in both equilibria, this spread of her posterior belief implies that $U^*(n_2) > U^*(n_1)$. This proves that $U^*(n)$ strictly increases with n.

6.14 Proof of Theorem 3

When n increases, the set of possible acceptance fractions in symmetric equilibria also changes. For example, when $n = 2$, the possible acceptance fractions are $\frac{1}{2}$ and 1. When $n = 3$, the possible acceptance fractions are $\frac{1}{3}$, $\frac{2}{3}$, and 1. To study equilibrium outcomes when $n \to \infty$, I take the following approach: fixing any positive rational number $\alpha \in (0,1]$, I focus on integers n such that α is a feasible acceptance fraction when there are n persuaders and I examine the set of symmetric equilibria as $n \to \infty$. I show that for all α, asymptotically, if there is a symmetric equilibrium in which the persuaders choose (x,y) and the acceptance fraction is α then y must be weakly higher than α to prevent profitable deviation. Moreover, if y is strictly higher than α, the decision maker learns the true state with probability 1 and the persuaders get the highest payoff they can get in any asymptotic symmetric equilibrium. Therefore, these symmetric equilibria with $y > \alpha$ are outcome-equivalent and they Pareto dominate all the other symmetric equilibria (namely, symmetric equilibria with $y = \alpha$).

The proof is structured as follows. Claim 1 lays the technical groundwork. Claim 2 proves the Theorem for symmetric binding equilibria in which $y = \bar{y}$. I then extend the proof to symmetric non-binding equilibria in which $y < \bar{y}$ in Claim 3. Note that, for any given α and n, while there is at most one symmetric binding equilibrium, there can be infinitely many symmetric non-binding equilibria.

Claim 1: Let α be an arbitrary rational number in $(0,1]$ and let n be a positive finite integer. *If an $n-$persuader symmetric test profile (x,y) induces acceptance fraction α and $y \geq \alpha$, then $x < \alpha$; moreover,* $\frac{dx}{dy} = 0$ when $y = \alpha$ and for any arbitrarily small $\varepsilon > 0$, $\frac{dx}{dy} \in (-\varepsilon, 0)$ when y is sufficiently close to α from above.

Proof. Let n be a positive finite integer such that α is a feasible acceptance fraction when n persuaders choose symmetric tests. In an $n-$persuader symmetric equilibrium with test profile (x,y) and acceptance fraction α, x and y must satisfy

$$\left(\frac{y}{x}\right)^{\alpha}\left(\frac{1-y}{1-x}\right)^{1-\alpha}=\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}. \tag{4}$$

Let $g(x,y)=\left(\frac{y}{x}\right)^{\alpha}\left(\frac{1-y}{1-x}\right)^{1-\alpha}$. Fixing y, the proof of Remark 2 shows that g is strictly decreasing when $x<\alpha$ and strictly increasing when $x>\alpha$. Moreover, $\lim_{x\to 0} g(x,y)=\infty$ and $g(y,y)=1$. Because $\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}>1$ for any finite n, the unique solution to the equation $g(x,y)=\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}$ is some $x<\alpha$.

The equation $g(x,y)=\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}$ implicitly defines x as a function of y. Differentiating both sides of this equation with respect to y yields

$$\frac{\partial g}{\partial x}\cdot\frac{dx}{dy}+\frac{\partial g}{\partial y}=0.$$

This implies that

$$\begin{aligned}
\frac{dx}{dy} &= -\frac{\partial g}{\partial y}\Big/\frac{\partial g}{\partial x} \\
&= \frac{(1-x)x}{(1-y)y}\cdot\frac{\alpha-y}{\alpha-x} \\
\frac{d^2x}{dy^2} &= -\frac{(1-x)x}{(\alpha-x)(1-y)^2y^2}\cdot\left[(y-\alpha)^2+\alpha-\alpha^2\right] \\
&< 0 \text{ and is continuous in } y.
\end{aligned}$$

These imply that $\frac{dx}{dy}=0$ when $y=\alpha$. For any arbitrarily small $\varepsilon>0$, $\frac{dx}{dy}\in(-\varepsilon,0)$ when y is sufficiently close to α from above. □

Claim 2: Fix $\bar{y}\in(0,1)$. Let α be an arbitrary rational number in $(0,1]$. Let there be n persuaders such that α is a feasible acceptance fraction when n persuaders choose symmetric tests. As $n\to\infty$, a symmetric binding equilibrium $(x,\bar{y})$ with acceptance fraction α exists only if $\bar{y}\geq\alpha$. If this equilibrium exists, then $\bar{y}>\alpha$ implies that the decision maker learns the true state with probability 1 and the persuaders' ex-ante expected utility is $\frac{1}{2}$; $\bar{y}=\alpha$ implies that the decision maker does not learn the true state and the persuaders' ex-ante expected utility is weakly less than $\frac{1}{2}$.

Proof. (a) As $n\to\infty$, the law of large numbers implies that the actual fraction of positive test results converges to the expected fraction of positive results. Since test results are conditionally independent, if the persuaders choose symmetric binding tests $(x,\bar{y})$, the expected fraction of positive results is equal to the probability that one persuader has a positive result, which is equal to $\bar{y}$ in state H and $x \leq \bar{y}$ in state L. This implies that if $(x,\bar{y})$ induces acceptance fraction $\alpha > \bar{y}$ then, asymptotically, the decision maker almost surely never chooses a_H and the persuaders' expected payoff is 0. Any persuader can profitably deviate to the test $(0,\bar{y})$ to increase the expected payoff from 0 to $\frac{1}{2}\bar{y}$. This proves that, as $n \to \infty$, a symmetric binding equilibrium $(x,\bar{y})$ with acceptance fraction α exists only if $\bar{y} \geq \alpha$.

(b) As $n \to \infty$, suppose that there exists a symmetric binding equilibrium $(x,\bar{y})$ with acceptance fraction α and $\bar{y} > \alpha$. By Claim 1, $x \leq \alpha$, which implies that $x < \bar{y}$. By the law of large numbers, this means that the actual fraction of positive results is almost surely equal to $\bar{y}$ in state H and strictly less than $\bar{y}$ in state L. By simply observing the actual fraction of positive results, the decision maker can perfectly distinguish state L from state H with probability 1. Therefore, the decision maker's ex-ante expected utility is $\bar{U}$ and the persuaders' ex-ante expected utility is $\frac{1}{2}$, the prior probability that the state is H.

(c) As $n \to \infty$, suppose that there exists a symmetric binding equilibrium $(x,\bar{y})$ with acceptance fraction α and $\bar{y} = \alpha$. Equation (4) implies that $x \to \alpha$ from below. Because x converges to $\bar{y}$, the decision maker cannot perfectly distinguish state H from state L by observing the actual fraction of positive results and her ex-ante expected utility is $U < \bar{U}$. The persuaders' ex-ante expected utility is

$$\begin{aligned}
V &= \lim_{n\to\infty} \frac{1}{2} \sum_{i=\alpha n}^{n} \binom{n}{i} \left[\bar{y}^i (1-\bar{y})^{n-i} + x^i (1-x)^{n-i}\right] \\
&= \lim_{n\to\infty} \frac{1}{2} \sum_{i=\alpha n}^{n} \binom{n}{i} \left[\alpha^i (1-\alpha)^{n-i} + x^i (1-x)^{n-i}\right] \\
&\leq \lim_{n\to\infty} \frac{1}{2} \sum_{i=\alpha n}^{n} \binom{n}{i} \left[\alpha^i (1-\alpha)^{n-i} + \alpha^i (1-\alpha)^{n-i}\right] \\
&= \lim_{n\to\infty} \sum_{i=\alpha n}^{n} \binom{n}{i} \left[\alpha^i (1-\alpha)^{n-i}\right] \\
&= \frac{1}{2}
\end{aligned}$$

□

The last equality holds because the median of a binomial distribution $B(n,\alpha)$ is αn as $n \to \infty$. This proves that the persuaders' ex-ante expected utility is weakly less than $\frac{1}{2}$, which is their ex-ante expected utility in (b).

Claim 3: Fix $\bar{y} \in (0,1)$. Let α be an arbitrary rational number in $(0,1]$. Let there be n persuaders such that α is a feasible acceptance fraction when n persuaders choose symmetric tests. As $n \to \infty$, a symmetric non-binding equilibrium (x,y) with $y < \bar{y}$ and acceptance fraction α exists only if $y \geq \alpha$. If this equilibrium exists and $y > \alpha$ as $n \to \infty$, the decision maker learns the true state with probability 1 and the persuaders' ex-ante expected utility is $\frac{1}{2}$; if $y \to \alpha$ as $n \to \infty$, the decision maker does not learn the true state and the persuaders' ex-ante expected utility is weakly less than $\frac{1}{2}$.

Proof. By simply replacing $\bar{y}$ with $y < \bar{y}$ in (a) and (b) of Claim 2's proof, one can show that as $n \to \infty$, a symmetric non-binding equilibrium (x,y) with $y < \bar{y}$ and acceptance fraction α exists only if $y \geq \alpha$; if this equilibrium exists and $y > \alpha$ as $n \to \infty$, the decision maker learns the true state with probability 1 and the persuaders' ex-ante expected utility is $\frac{1}{2}$.

Now, suppose that there is a sequence of symmetric non-binding equilibria (x_n, y_n) such that $y_n < \bar{y}$ for all n and $y_n \to \alpha$ from *above* as $n \to \infty$. Equation (4) and Claim 1 imply that $x_n \to \alpha$ from below. Because both x_n and y_n converge to α, the decision maker cannot perfectly distinguish state H from state L by observing the actual fraction of positive results and her ex-ante expected utility is $U < \bar{U}$.

The persuaders' payoff is equal to the unconditional probability that at least αn test results are positive. The number of positive test results follows a binomial distribution $B(n, y_n)$ in state H and $B(n, x_n)$ in state L. By the de Moivre–Laplace theorem, as $n \to \infty$, the probability mass function of the number of successes from a binomial distribution $B(n, y_n)$ (respectively, $B(n, x_n)$) converges to the probability density function of the normal distribution $N(ny_n, ny_n(1-y_n))$ (respectively, $N(nx_n, nx_n(1-x_n))$) as long as $y_n,\ x_n$ do not converge to 0 or 1, which is true because they converge to $\alpha \in (0, \bar{y})$. Therefore, as $n \to \infty$, the persuaders' ex-ante expected utility converges to

$$
\begin{aligned}
V &= \frac{1}{2}\{[1-\Phi(\alpha n, ny_n, ny_n(1-y_n))] + [1-\Phi(\alpha n, nx_n, nx_n(1-x_n))]\} \\
&= \frac{1}{2}\left\{\frac{1}{2}\left[1-erf\left(\frac{\alpha n - ny_n}{\sqrt{2ny_n(1-y_n)}}\right)\right] + \frac{1}{2}\left[1-erf\left(\frac{\alpha n - nx_n}{\sqrt{2nx_n(1-x_n)}}\right)\right]\right\} \\
&= \frac{1}{2} + \frac{1}{4}\left[erf\left(\frac{ny_n - \alpha n}{\sqrt{2ny_n(1-y_n)}}\right) - erf\left(\frac{\alpha n - nx_n}{\sqrt{2nx_n(1-x_n)}}\right)\right]
\end{aligned}
$$

where

$$\Phi\left(x,\mu,\sigma^2\right)=\frac{1}{2}\left[1+erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

is the c.d.f. of a normal distribution $N\left(\mu,\sigma^2\right)$ and erf is the error function.

Note that $y_n\to\alpha$ from above and $x_n\to\alpha$ from below means that $x_n<\alpha<y_n$ for any finite n. This implies that

$$\begin{aligned}\frac{ny_n-\alpha n}{\sqrt{2ny_n\left(1-y_n\right)}} &> 0\\ \frac{\alpha n-nx_n}{\sqrt{2nx_n\left(1-x_n\right)}} &> 0\\ \frac{ny_n-\alpha n}{\sqrt{2ny_n\left(1-y_n\right)}}\Big/\frac{\alpha n-nx_n}{\sqrt{2nx_n\left(1-x_n\right)}} &= \frac{y_n-\alpha}{\alpha-x_n}\cdot\sqrt{\frac{x_n\left(1-x_n\right)}{y_n\left(1-y_n\right)}}\end{aligned}$$

Recall from Claim 1 that $\frac{dx}{dy}=\frac{(1-x)x}{(1-y)y}\cdot\frac{\alpha-y}{\alpha-x}$. Therefore,

$$\frac{ny_n-\alpha n}{\sqrt{2ny_n\left(1-y_n\right)}}\Big/\frac{\alpha n-nx_n}{\sqrt{2nx_n\left(1-x_n\right)}} = -\frac{dx_n}{dy_n}\cdot\sqrt{\frac{y_n\left(1-y_n\right)}{x_n\left(1-x_n\right)}}.$$

By Claim 1, for any arbitrarily small $\varepsilon>0$, $-\frac{dx}{dy}\in(0,\varepsilon)$ when y_n is sufficiently close to α.

Note that as $x_n\to\alpha$ and $y_n\to\alpha$,

$$\sqrt{\frac{y_n\left(1-y_n\right)}{x_n\left(1-x_n\right)}} \to 1.$$

Therefore, let $\varepsilon\in\left(0,\frac{1}{2}\right)$,

$$0<-\frac{dx_n}{dy_n}\cdot\sqrt{\frac{y_n\left(1-y_n\right)}{x_n\left(1-x_n\right)}}<\varepsilon\cdot 1<\frac{1}{2}<1$$

when y_n is sufficiently close to α. This implies that, as $n\to\infty$ and $y_n\to\alpha$ from above,

$$
\begin{aligned}
\frac{ny_n-\alpha n}{\sqrt{2ny_n(1-y_n)}}/\frac{\alpha n-nx_n}{\sqrt{2nx_n(1-x_n)}} &< 1 \\
\frac{ny_n-\alpha n}{\sqrt{2ny_n(1-y_n)}} &< \frac{\alpha n-nx_n}{\sqrt{2nx_n(1-x_n)}}
\end{aligned}
$$

Because the error function is strictly increasing,

$$
\begin{aligned}
erf\left(\frac{ny_n-\alpha n}{\sqrt{2ny_n(1-y_n)}}\right)-erf\left(\frac{\alpha n-nx_n}{\sqrt{2nx_n(1-x_n)}}\right) &\leq 0 \\
V &\leq \frac{1}{2}
\end{aligned}
$$

Finally, fixing α, if y_n converges to α from *below*, x_n converges to α from below, too. The probability that the actual number of positive results exceeds αn is weakly lower than $\frac{1}{2}$ in both states because the median of $B(n,y_n)$ is $ny_n \leq \alpha$ and the median of $B(n,x_n)$ is $nx_n \leq \alpha$ as $n \to \infty$. Therefore, $V \leq \frac{1}{2}$ in this case as well. This proves Claim 3. □

Claim 2 and Claim 3 prove that, among all symmetric equilibria, the Pareto optimal ones are those in which the test's true-positive probability (y) is strictly higher than the acceptance fraction. In these equilibria, the decision maker obtains $U=\bar{U}$ because she almost surely learns the true state and the persuaders obtain $V=\frac{1}{2}$. In all the other symmetric equilibria, the decision maker obtains $U<\bar{U}$ and the persuaders obtain $V \leq \frac{1}{2}$.

Finally, Theorem 2 (existence of symmetric binding equilibrium with acceptance fraction $\frac{1}{n}$) implies that the truth-revealing Pareto optimal symmetric equilibrium with $y>\alpha$ always exists. In this symmetric binding equilibrium, the acceptance fraction is $\alpha=\frac{1}{n}\to 0$ as n increases and the true-positive probability is $\bar{y}>0$

6.15 Proof of Theorem 4

Each persuader's payoff in the symmetric binding equilibrium with $\alpha=\frac{1}{n}$ is

$$
\begin{aligned}
V_n &= \frac{1}{2}\left[1-(1-\bar{y})^n+1-(1-x^*)^n\right] \\
&= 1-\frac{1}{2}\left[(1-\bar{y})^n+(1-x^*)^n\right]
\end{aligned}
$$

where x^* is a function of $\bar{y}$ that satisfies

$$g(x^*,\bar{y}) \equiv \left(\frac{\bar{y}}{x^*}\right)\left(\frac{1-\bar{y}}{1-x^*}\right)^{n-1} = \frac{p_d}{1-p_d}.$$

The proof of Remark 2 has shown that $x^* < \frac{1}{n}$ for all $\bar{y}$, p_d, and $n < \infty$. Claim 1 in the proof of Theorem 3 has shown that

$$\begin{aligned}
\frac{dx^*}{d\bar{y}} &= -\frac{\partial g}{\partial \bar{y}}/\frac{\partial g}{\partial x^*} \\
&= \frac{(1-x^*)x^*}{(1-\bar{y})\,\bar{y}} \cdot \frac{1-n\bar{y}}{1-nx^*} \\
& \begin{cases} >0 & \text{when } \bar{y} < \frac{1}{n} \\ =0 & \text{when } \bar{y} = \frac{1}{n} \\ <0 & \text{when } \bar{y} > \frac{1}{n} \end{cases} \\
\frac{d^2x^*}{d\bar{y}^2} &= -\frac{(1-x^*)\,x^*}{(\frac{1}{n}-x^*)\,(1-\bar{y})^2\,\bar{y}^2} \cdot \left[\left(\bar{y}-\frac{1}{n}\right)^2 + \frac{1}{n} - \left(\frac{1}{n}\right)^2\right] \\
&< 0
\end{aligned}$$

Because $V_{solo} = \frac{\bar{y}}{2p_d}$ and $x^*\,(1-x^*)^{n-1} = \frac{1-p_d}{p_d}\bar{y}\,(1-\bar{y})^{n-1}$,

$$\begin{aligned}
\frac{d\,(V_n - V_{solo})}{d\bar{y}} &= \frac{n}{2}\left[(1-\bar{y})^{n-1} + (1-x^*)^{n-1}\,\frac{dx^*}{d\bar{y}}\right] - \frac{1}{2p_d} \\
&= \frac{n}{2}\,(1-\bar{y})^{n-1}\left[1 + \frac{1-p_d}{p_d} \cdot \frac{(1-x^*)}{(1-\bar{y})} \cdot \frac{1-n\bar{y}}{1-nx^*}\right] - \frac{1}{2p_d} \\
\frac{d^2\,(V_n - V_{solo})}{d\bar{y}^2} &= \frac{n}{2}\left[-(n-1)\,(1-\bar{y})^{n-2}\right. \\
&\quad -(n-1)\,(1-x^*)^{n-2}\,\frac{dx^*}{d\bar{y}} \\
&\quad \left. +(1-x^*)^{n-1}\,\frac{d^2x^*}{d\bar{y}^2}\right]
\end{aligned}$$

When $\bar{y} \to 0$, both V_n and V_{solo} converge to 0 so $V_n - V_{solo} \to 0$. The first derivative of $V_n - V_{solo}$ converges to

$$
\begin{aligned}
\frac{d\left(V_n - V_{solo}\right)}{d\bar{y}} &\overset{\bar{y}\to 0}{\longrightarrow} \frac{n}{2}\left(1+\frac{1-p_d}{p_d}\right)-\frac{1}{2p_d} \\
&\geq 1+\frac{1-p_d}{p_d}-\frac{1}{2p_d} \\
&= \frac{1}{2p_d} \\
&> 0
\end{aligned}
$$

and the second derivative $\frac{d^2(V_n - V_{solo})}{d\bar{y}^2} < 0$ when $\bar{y} < \frac{1}{n}$. In other words, when $\bar{y}$ is close to 0, $V_n - V_{solo}$ is positive, strictly increasing, and strictly concave.

When $\bar{y}$ is relatively large, the slope and curvature of $V_n - V_{solo}$ is less clear, but $\frac{d^2(V_n - V_{solo})}{d\bar{y}^2} \geq 0$ always implies $\frac{d(V_n - V_{solo})}{d\bar{y}} < 0$ because

$$
\begin{aligned}
\frac{d^2\left(V_n - V_{solo}\right)}{d\bar{y}^2} &\geq 0 \\
\Rightarrow -\left(1-\bar{y}\right)^{n-2} - \left(1-x^*\right)^{n-2}\frac{dx^*}{d\bar{y}} &> 0 \\
\left(1-\bar{y}\right)^{n-2} + \left(1-x^*\right)^{n-2}\frac{dx^*}{d\bar{y}} &< 0 \\
\left(1-\bar{y}\right)^{n-2}\left(1-x^*\right) + \left(1-x^*\right)^{n-1}\frac{dx^*}{d\bar{y}} &< 0 \\
\left(1-\bar{y}\right)^{n-2}\left(1-\bar{y}\right) + \left(1-x^*\right)^{n-1}\frac{dx^*}{d\bar{y}} &< 0 \\
\left(1-\bar{y}\right)^{n-1} + \left(1-x^*\right)^{n-1}\frac{dx^*}{d\bar{y}} &< 0 \\
\frac{d\left(V_n - V_{solo}\right)}{d\bar{y}} &< 0
\end{aligned}
$$

In other words, $V_n - V_{solo}$ must be strictly decreasing whenever it is not concave. This means that once $V_n - V_{solo}$ starts to decrease after a certain value of $\bar{y}$, it remains strictly decreasing.

When $\bar{y} = 1$, V_{solo} is the maximum of the persuaders' ex-ante expected utility (Kamenica and Gentzkow, 2011). Because a two-point posterior belief distribution (the decision maker either believes $\Pr(H) = 0$ or $\Pr(H) = p_d$) is required to achieve V_{solo} and the symmetric binding equilibrium with $\alpha = \frac{1}{n}$ does not induce this distribution, $V_n - V_{solo} < 0$ when $\bar{y} = 1$.

To summarize, $V_n - V_{solo}$ is initially strictly increasing and concave in $\bar{y}$ for sufficiently small $\bar{y}$, and strictly decreasing whenever it is not concave. On the two extreme ends of the domain, $V_n - V_{solo} \to 0$ from above when $\bar{y} \to 0$ and $V_n - V_{solo} < 0$ when $\bar{y} = 1$. This implies that $V_n - V_{solo}$ must be first strictly increasing and then strictly decreasing on the domain of $\bar{y}$, and there exists a unique threshold $\bar{y}^*$ such that $V_n - V_{solo} > 0$ if and only if $\bar{y} < \bar{y}^*$.

Finally, to show that this threshold $\bar{y}^*$ is above p_d, note that when $\bar{y} = p_d$,

$$V_{solo} = \frac{1}{2}.$$

x^* satisfies

$$\begin{aligned}
\left(\frac{p_d}{x^*}\right)\left(\frac{1-p_d}{1-x^*}\right)^{n-1} &= \frac{p_d}{1-p_d} \\
x^*\left(1-x^*\right)^{n-1} &= \left(1-p_d\right)^n
\end{aligned}$$

which means that

$$\begin{aligned}
V_n &= 1-\frac{1}{2}\left[\left(1-p_d\right)^n+\left(1-x^*\right)^n\right] \\
&= 1-\frac{1}{2}x^*\left(1-x^*\right)^{n-1}\left(1+\frac{1-x^*}{x^*}\right) \\
&= 1-\frac{1}{2}\left(1-x^*\right)^{n-1} \\
&> \frac{1}{2} = V_{solo}
\end{aligned}$$

Therefore, $V_n > V_{solo}$ for all $0 < \bar{y} \le p_d$, which means that $\bar{y}^* > p_d$. This completes the proof.

6.16 Proof of Theorem 5

When $\bar{y} < 1$, as the number of informative persuaders goes to infinity, the decision maker behaves as if she knows the true state in all Pareto optimal symmetric equilibria - i.e., she chooses a_H if and only if the state is H with probability 1 (Theorem 3). Therefore, each persuader's ex-ante expected utility V_∞ is equal to $\Pr(H) = \frac{1}{2}$. Because $V_{solo} = \frac{\bar{y}}{2p_d}$,

$V_\infty > V_{solo}$ if and only if $\bar{y} < p_d$.

6.17 Proof of Lemma 2

Without loss of generality, consider a non-beneficial test profile with $n > 1$ informative tests. Based on the "only if" part of the proof in Proposition 1, the fact that this test profile is non-beneficial implies that the decision maker chooses a_H if and only if all n test results are positive. Let y_0 denote the probability that all n results are positive in state H. Let x_0 denote the probability that all n results are positive in state L. Let V_0 denote each persuader's expected utility.

Next, let there be only one informative persuader whose test design is (x_0, y_0). The decision maker chooses a_H if and only if the test result is positive. The distribution of the decision maker's actions conditional on the state under this test is the same as that under the previous test profile with n tests. Therefore, the single persuader's expected utility is also equal to V_0. This shows that a single persuader can replicate V_0 from any non-beneficial test profile.

Finally, because V_{solo} is the maximum of all feasible payoffs for a single persuader, and the payoff from any non-beneficial test profile is feasible, V_{solo} is the maximum payoff that any persuader can achieve in any non-beneficial test profile regardless of n.

6.18 Proof of Lemma 3

If there exists an equilibrium with only one informative persuader, then this persuader's test design must be $\left(\frac{1-p_d}{p_d}\bar{y}, \bar{y}\right)$ and his payoff is $V_{solo} = \frac{\bar{y}}{2p_d}$. The decision maker chooses a_H if and only if the test result is positive, in which case she is indifferent between a_H and a_L; her expected utility is $U = 0$.

Without loss of generality, let persuader 1 be the only informative persuader. Suppose that there is a unilateral deviation by (the currently uninformative) persuader 2 to an informative test. If this deviation is profitable, then the decision maker's new acceptance set must be $\{\{2\}, \{1,2\}\}$ - i.e., she wants to choose a_H if and only if persuader 2's test result is positive. $\{1\}$ does not belong in the acceptance set because a positive result from persuader 1 on its own merely makes the decision maker indifferent, so the decision maker must strictly prefer a_L when she sees a negative result from persuader 2. $\{2\}$ belongs in the acceptance set because if it did not, then persuader 2 merely deviates from one non-beneficial test profile to another non-beneficial profile. However, by Lemma 2, when persuader 1 is the only informative persuader, he can achieve any payoff associated with any non-beneficial test profile with a single test, so a deviation that induces a different non-beneficial test profile cannot be more profitable. This is why a profitable deviation must induce a bigger acceptance set than $\{\{1,2\}\}$.

Given that a deviation from persuader 2 induces the acceptance set $\{\{2\},\{1,2\}\}$, the most profitable deviation of this type is one that leaves the decision maker just indifferent when (1) persuader 1's test result is negative and (2) persuader 2's test result is positive. Let $\left(x^{'},y^{'}\right)$ denote the deviation, then

$$\frac{y^{'}}{x^{'}}\cdot\frac{1-\bar{y}}{1-\frac{1-p_d}{p_d}\bar{y}} = \frac{p_d}{1-p_d}.$$

Since the persuader's payoff strictly increases in $x^{'}$ and $y^{'}$, and $x^{'}<y^{'}\leq\bar{y}$, the best deviation for persuader 2 is

$$\begin{aligned} y^{'} &= \bar{y} \\ x^{'} &= \frac{\bar{y}}{\frac{p_d}{1-p_d}}\cdot\frac{1-\bar{y}}{1-\frac{1-p_d}{p_d}\bar{y}}=\frac{\bar{y}(1-\bar{y})}{\frac{p_d}{1-p_d}-\bar{y}} \end{aligned}$$

Following the deviation, persuader 2's payoff becomes

$$\begin{aligned} V^{'} &= \frac{1}{2}\left(x^{'}+y^{'}\right) \\ &= \frac{1}{2}\left[\frac{\bar{y}(1-\bar{y})}{\frac{p_d}{1-p_d}-\bar{y}}+\bar{y}\right] \end{aligned}$$

This deviation is profitable if and only if

$$\begin{aligned} V^{'} &> V_{solo} \\ \frac{\bar{y}(1-\bar{y})}{\frac{p_d}{1-p_d}-\bar{y}}+\bar{y} &> \frac{\bar{y}}{p_d} \\ \frac{\bar{y}(1-\bar{y})}{\frac{p_d}{1-p_d}-\bar{y}} &> \frac{1-p_d}{p_d}\bar{y} \\ \frac{1-\bar{y}}{\frac{p_d}{1-p_d}-\bar{y}} &> \frac{1-p_d}{p_d} \\ \left(1-\frac{1-p_d}{p_d}\right)\bar{y} &< 0 \end{aligned}$$

However, because $\frac{1-p_d}{p_d} < 1$, the inequality above never holds. Therefore, a unilateral deviation from persuader 2 is never profitable. This proves that there is always a non-beneficial asymmetric equilibrium with only one informative persuader and his test is $\left(\frac{1-p_d}{p_d}\bar{y}, \bar{y}\right)$.

By Lemma 2, the persuaders' payoff in this asymmetric equilibrium is weakly higher than their payoff in any other non-beneficial equilibrium.

6.19 Proof of Theorem 6

By Lemma 3, the highest payoff the persuaders can achieve in any (symmetric or asymmetric) non-beneficial equilibrium is V_{solo}. By Theorem 4, when $n \geq 2$, for any $p_d \in \left(\frac{1}{2}, 1\right)$, there exists a $\bar{y}^* \in (p_d, 1)$ such that V_{solo} is lower than the persuaders' payoff in some beneficial equilibrium if $\bar{y} < \bar{y}^*$. By Theorem 5, when $n \to \infty$, V_{solo} is lower than the persuaders' payoff in a state-revealing equilibrium if $\bar{y} < p_d$. This leads to the conclusion in Theorem 6.

6.20 Proof of Section 4.C (sequential persuaders)

Suppose that n persuaders choose their tests sequentially. Each persuader, as well as the decision maker, observes the test designs and results of all previous persuaders.

By backward induction, suppose that it is the n^{th} persuader's turn to choose and the public belief is $\Pr(H) = p_n$. If p_n is already higher than or equal to p_d, persuader n's optimal strategy is to do nothing (e.g. he can choose the uninformative test $(x,y) = (1,1)$) because the current belief already induces a_H with certainty. If $p_n < p_d$, persuader n's optimal test is $(x_n, \bar{y}) = \left(\bar{y}\frac{p_n(1-p_d)}{p_d(1-p_n)}, \bar{y}\right)$, so that $\Pr(H \mid positive) = p_d$. The expected payoff of persuader n is $V_n(p_n) = 1$ when $p_n \geq p_d$ and $V_n(p_n) = \bar{y}\frac{p_n}{p_d} < 1$ when $p_n < p_d$.

Now, suppose that it is the $n-1^{th}$ persuader's turn to choose and the public belief is $\Pr(H) = p_{n-1}$. Once again, this persuader chooses an informative test only when $p_{n-1} < p_d$. As calculated earlier, if his test result induces posterior belief $p_n \geq p_d$, persuader $n-1$'s expected payoff is 1; if his result induces $p_n < p_d$, persuader $n-1$'s expected payoff is $\bar{y}\frac{p_n}{p_d}$, which is linear in p_n and strictly lower than 1. Therefore, if $p_{n-1} < p_d$, the optimal test maximizes the chance of inducing $p_n \geq p_d$ by making the decision maker's posterior belief equal to exactly p_d when the test result is positive. In other words, the $n-1^{th}$ persuader behaves as if he is the last persuader in the game. When $p_{n-1} < p_d$, the $n-1^{th}$

persuader's expected payoff is $V_{n-1}(p_{n-1}) = \bar{y}\frac{p_{n-1}}{p_d} + \left(1 - \bar{y}\frac{p_{n-1}}{p_d}\right) V_n(p_n(p_{n-1}, -)) < 1$, where $p_n(p_{n-1}, -)$ is the posterior belief following persuader $n-1$'s negative test result.

By induction, every persuader behaves in the same way: if the current belief weakly exceeds p_d, the next persuader does nothing. If the current belief is below p_d, the next persuader acts as if he were the last persuader and chooses a test $(x, \bar{y})$ such that the posterior belief after a positive result is p_d. The decision maker's belief update stops as soon as she observes one positive result, in which case she chooses a_H. She chooses a_L if and only if all test results are negative. The decision maker's ex-ante expected utility is $U = 0$ because her posterior belief never strictly exceeds p_d. In equilibrium, the decision maker either chooses her default action a_L or is indifferent between switching to a_H and staying with her default action. Therefore, U is equal to her payoff when she always stays with her default action a_L, which is 0.

While the decision maker does not benefit in this sequential game, the persuaders benefit greatly when there are many of them. Note that if the current belief is $p_i < p_d$, the optimal test for persuader i is $(x_i^*, \bar{y})$, where $x_i^* = \bar{y}\frac{p_i(1-p_d)}{p_d(1-p_i)}$, and the expected payoff is

$$V_i(p_i) = \bar{y}\frac{p_i}{p_d} + \left(1 - \bar{y}\frac{p_i}{p_d}\right) V_{i+1}(p_{i+1}(p_i, -)),$$

where

$$p_{i+1}(p_i, -) = \frac{(1-\bar{y})\, p_i}{(1-\bar{y})\, p_i + \left[1 - x_i^*(p_i)\right](1-p_i)}$$

is the posterior belief when i's test result is negative.

As $n \to \infty$, given prior belief $p_1 < p_d$, the expected payoff of persuader 1 converges to $V = \frac{p_1}{p_d}$, which is equal to a persuader's highest attainable payoff when exogenous noise disappears (i.e., $\bar{y} = 1$). Intuitively, with infinitely many sequential persuaders, the decision maker's posterior belief is either equal to p_d or converging to 0 (the latter is the belief after infinitely many negative results). This two-point posterior belief distribution is identical to the optimal feasible posterior belief distribution when persuaders are free from exogenous noise.

6.21 Example: exogenous bounds on the false-positive probability (Section 4.D)

Here, I use a few examples to explain the role of a lower bound on x in details. Suppose that there exists some $\underline{x} > 0$ such that each persuader i must choose $\Pr(positive|L) = x_i \geq \underline{x}$. When $\underline{x}$ is sufficiently high, tests that induce low acceptance fractions are no longer feasible. This has two implications: beneficial equilibria with low acceptance fractions disappear, and profitable deviations disappear, too.

To see how a high $\underline{x}$ can eliminate equilibria with low acceptance fractions, note that the false-positive probability x^* in equilibrium is negatively associated with the acceptance fraction. For example, Table 3 lists the value of $x^*(\alpha)$ in all symmetric binding equilibria with different acceptance fractions α when $n = 3$, $p_d = 0.6$ or 0.8, and $\bar{y} = 0.95$[9]. Note that, for any α, if $\underline{x} > x^*(\alpha)$, no symmetric equilibria with acceptance fraction lower than or equal to α exist. For example, when $n = 3$, $p_d = 0.6$, and $\bar{y} = 0.95$, if $\underline{x} \in (0.0016, 0.19)$ then only symmetric equilibria with $\alpha \geq \frac{2}{3}$ exist.

To see how a high $\underline{x}$ can eliminate profitable deviation, note that any profitable deviation is associated with a lowered acceptance fraction and, hence, a lowered false-positive probability (Lemma 1). For example, if a profitable deviation from the non-beneficial symmetric test profile exists, then the false-positive probability associated with it is lower than or equal to the value $x'(c = 1)$ given in Lemma 4. Table 4 lists the values of $x'(c = 1)$ for different values of p_d and $\bar{y}$ when $n = 3$. When $\underline{x}$ is lower than these values, the non-beneficial symmetric equilibrium does not exist. When $\underline{x}$ exceeds these values, profitable deviations from the non-beneficial binding test are no longer feasible; hence, non-beneficial symmetric equilibrium exists. The same results can be applied to any beneficial equilibrium, too. In general, when $\underline{x}$ is sufficiently high, profitable deviations are rarer, and it is easier for a test profile to be an equilibrium.

		Acceptance fraction		
		$\alpha = \frac{1}{3}$	$\alpha = \frac{2}{3}$	$\alpha = 1$
p_d	0.6	0.0016	0.19	0.83
	0.8	0.00058	0.11	0.60

Table 3: False-positive probabilities (x^*) in symmetric binding equilibria when $n = 3$ and $\bar{y} = 0.95$.

[9] $\bar{y}$ is chosen to be higher than 0.92 so that all symmetric binding equilibria exist.

		$\bar{y}$		
		0.6	0.7	0.8
p_d	0.6	0.385	0.413	0.405
	0.8	0.153	0.149	0.128

Table 4: Highest false-positive probabilities $\left(x^{'}(c=1)\right)$ in a profitable deviation from a non-beneficial symmetric test profile when $n = 3$.

Overall, the effect of a lower bound $\underline{x}$ on the number of equilibria is ambiguous. On the one hand, the disappearance of profitable deviations can lead to more equilibria. On the other hand, a lower bound on x can also lead to the disappearance of equilibria with low acceptance fractions. The exact effect of $\underline{x}$ depends on the parameter values.

Finally, an upper bound on x prevents high false-positive probabilities in the test designs. Therefore, it eliminates equilibria with poorly informative tests and small acceptance sets, such as the non-beneficial equilibria. As illustrated in Table 3, if the upper bound on x is lower than the numbers in the third column, then the non-beneficial symmetric equilibrium disappears, while other beneficial equilibria continue to exist.

6.22 Proof of Section 4.F (non-binary test results)

The binary assumption of test results is not without loss of generality when there are multiple persuaders, but relaxing this assumption only strengthens this paper's results.

For example, suppose that there is no exogenous bound on the test design. Also suppose that $M_1 = \{positive,\ negative\}$ for persuader 1 and he picks $\Pr(positive\,|\,L) \approx 0.067$, $\Pr(positive\,|\,H) \approx 0.5$. Assume that $M_2 = \{A, B, C\}$ for persuader 2. Consider the following strategy: $\Pr(A|H) = \frac{3}{5}$, $\Pr(A|L) \approx 0.08$, $\Pr(B|H) = \frac{2}{5}$, $\Pr(B|L) \approx 0.75$, $\Pr(C|H) = 0$, $\Pr(C|L) \approx 0.17$. These numbers are chosen so that the decision maker is just indifferent when she sees $(negative,\ A)$ or $(positive,\ B)$. Hence, the persuader's payoff is the unconditional probability $\Pr(A) + \Pr(positive,\ B) \approx 0.465$. This value is higher than the highest payoff that persuader 2 could get if M_2 is simply $\{positive,\ negative\}$.[10] Therefore, there does not exist any feasible test under $M_2 = \{positive,\ negative\}$ that is outcome-equivalent to the proposed test with $M_2 = \{A, B, C\}$.

However, any non-beneficial equilibrium with $M \subseteq \mathbb{R}$ is outcome-equivalent to some equilibrium with $M = \{positive,\ negative\}$. To see why, note that, in equilibria with $M \subseteq \mathbb{R}$

[10]When $M_2 = \{positive,\ negative\}$, persuader 2's best response to persuader 1's test design is to mimic the same test design with conditional probabilities $(0.067, 0.5)$, which yields an expected payoff of 0.44.

and $U = 0$, the decision maker chooses a_H with positive probability and is always indifferent when choosing a_H. This implies that each persuader i's test has a most positive message m_i^* with the highest $\frac{\Pr(m_i^*|H)}{\Pr(m_i^*|L)}$ among all possible messages, and the decision maker chooses a_H if and only if the realized messages are $\{m_1^*, m_2^*, ..., m_n^*\}$. In this case, it is possible to construct an outcome-equivalent test with $M' = \{positive, negative\}$ for each persuader i. Let $\Pr(positive|\omega) = \Pr(m_i^*|\omega)$ and $\Pr(negative|\omega) = \sum_{m \neq m_i^*} \Pr(m|\omega)$. The distribution of the decision maker's actions conditional on the state under the new tests is the same as that under the original tests.

Therefore, relaxing the binary restriction on M does not change the set of non-beneficial equilibrium outcomes with $U = 0$; it only increases the number of beneficial equilibrium outcomes with $U > 0$. Since beneficial equilibria with $U > 0$ already exist when M is binary, they continue to exist when M is larger; if non-beneficial equilibria do not exist when M is binary, they still do not exist when M is larger; if persuaders prefer some beneficial equilibrium over a non-beneficial equilibrium when M is binary, they continue to exhibit this preference when M is larger. Hence, the results of this paper are robust when the binary restriction of M is relaxed.

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